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EDITORIAL POLICY

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Nationwide Mass Appraisal Modeling in China: Feasibility Analysis for Scalability Given Ad Valorem Property Tax Reform

BY PEADAR DAVIS, PH.D., MRICS; MICHAEL MCCORD, PH.D.; PAUL BIDANSET; AND MARGIE CUSACK

Abstract

The Chinese government has stated its intention of introducing an annual property tax since 2003, but, while selecting six pilot cities for experimenting with the viability of a mass appraisal system rollout, has not yet adopted this policy. The Shenzhen Center for Assessment and Development of Real Estate was founded to facilitate the process of piloting the viability of property taxes — an initiative that coincided with the Lincoln Institute of Land Policy's initial involvement in China in 2003 (with the International Property Tax Institute [IPTI], ESRI Canada, and others) — and to provide expertise in topics ranging from property tax and municipal finance to public land management and land expropriation. The long-standing intention to roll out property tax, allied with significant capacity building, begs the questions, why has there not been more progress to date, and are there any fundamental barriers to policy adoption? This paper seeks to contribute to understanding this issue by assessing the feasibility of creating computer-assisted mass appraisal (CAMA) and automated valuation models (AVMs) in China and their respective capability to conform to IAAO valuation standards, with implications for scalability across national and regional markets.

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Introduction

The vast restructuring of the Chinese economy, urbanization, and social processes (Ma 2004), in tandem with a series of sweeping housing reforms that have taken place since 1978, has fundamentally transformed the nature of Chinese cities. These reforms have engendered a gradual shift toward a market system, with new institutions being established to enable the decentralized, monetized, and privatized allocation of housing (Tang, Haila, and Wong 2006). As a consequence of these processes, Li (2005) estimated that more than 80 percent of public housing was sold to existing tenants over the two decades since the introduction of policy reforms, and Tan et al. (2005) indicated that this has led to the development of vibrant resale markets. Despite these evolving policy reforms, Li (2005) suggested that different regions and cities have proceeded at a different pace, which makes the equitability of (future) policy uncertain.

The economic prosperity in China over the past two decades, and particularly that associated with house price inflation, has resulted in the Chinese government adopting a series of policies directed at the housing market. These policies include home purchase restrictions and the intention to introduce a recurrent property tax (Du and Zhang 2015). The home-purchase restriction was first introduced in Beijing in 2010 and progressively implemented in most major cities throughout China. This tax policy initiative prohibits resident households from buying more than two homes and nonresident households from buying more than one home. In terms of property tax, there has been only limited progress.

Pilot property tax programs were implemented in Shanghai and Chongqing in 2011; in Shanghai the property tax targets second homes; and the property tax enacted in Chongqing is mainly levied on high-end homes. These pilot programs have generated heated debate on inequity and inequality, related to the potential effects of the diverse distribution of income and urbanization-related issues. This is further compounded by the lack of a uniform designation of a proper property tax reform process (Cao and Hu 2016).

Therefore, despite considerable technical progress, property tax reform in China continues to be challenging. There remains considerable opposition to reforms from investors and local government officials alike, propagated by concerns that they may curb infrastructure investment, local gross domestic product (GDP) growth, and development. This complex economic debate is exacerbated by confusion and misunderstanding (Man 2011). In addition, there has been limited and piecemeal development of the necessary laws, regulations, and assessment standards necessary for policy enactment. In actuality, almost every aspect of the proposed property tax system — the nature and specification of a tax base, exemptions, assessment and administration systems, rate-setting powers, and allocation of the tax revenues (Liu 2018) — is as yet unspecified and up for debate. Against this rather unpromising state of affairs, annual tax assessed on property value is still viewed as an efficient revenue resource that can reduce the dependency on land transfer fees — the dependence on which has fueled rises in property prices. The revenue argument is strengthened further by the reality that housing policy changes and land fee restrictions have resulted in a sharp decline in land transfer fees across 130 cities (China Index Institute 2012), highlighting the need to raise more sustainable revenue in the longer term. It is suggested by theory, and accepted in policy circles, that deployment of a property tax system can offer an efficient, equitable, and sustainable source of municipal revenue, while providing a check mechanism on property price inflation.

Nonetheless, although the Chinese government has long considered the introduction of an annual property tax, it has not yet been fully deployed, with taxes only at the point of sale (Nunlist 2017). Therefore, while there is some progress in technical research and, to an extent, property tax reform is gaining awareness if not necessarily acceptance in the public psyche, it remains embryonic, and if it is to be become a major source of public revenue, considerable support is required (Man 2011). In this context and in furthering this agenda, as attested to by Nunlist (2017), the Lincoln Institute of Land Policy, via the Peking University–Lincoln Institute Center for Urban Development and Land Policy (PLC) and along with IPTI and ESRI Canada (particularly the noble efforts of Lomax and Yuen) exemplifies the commitment of the international assessment community to provide international and domestic expertise, via commissioning research and demonstration projects on property taxation and related topics. This commitment has encompassed various projects, such as a pilot demonstration that established a CAMA system for the financial district of Beijing and several other city implementations of CAMA, in anticipation of a future property tax.

There is some evidence of gearing-up for the operationalization of CAMA systems, as evidenced by the housing registration system being developed to prepare for future property tax reform (Cao and Hu 2016), which is likely to form part of an enabling environment. However, the rollout of mass appraisal practice has been disjointed and limited, further compounded by the differing valuation approaches adopted to date. The introduction of pilot schemes in a number of cities using dissimilar methods has also, arguably, impinged upon the implementation of a more unified approach. This lack of unified approach has become a more pressing matter in light of continued house price appreciation, tight local government budgets, and the rising income gap (Cao and Hu 2016). Central Government in China has not yet decided whether to impose the property tax at a nationwide level or whether this will take the form of a uniform rate or a regional rate decided by local government. There is still a long way to go in this regard, and questions remain about the viability of undertaking nationwide property tax appraisal.

This paper therefore explores the nature of a nationwide rollout of mass appraisal in China and assesses whether mass appraisal models developed for China would be scalable and would conform to international benchmarks.

Property Tax in China

While China has no existing, comprehensive, modern ad valorem recurrent property tax as would be recognized in the traditional sense, there are a variety of taxes in use. Indeed, China has eight different taxes on property (SAT 2012; Hong 2012), five of which are related to real estate properties and account for approximately 22 percent of local tax revenues (Liu 2018). Three of the taxes can be classified as a property tax: the House Property Tax, the Urban and Township Land Use Tax, and the Tax on the Use of Arable Land. In the collective sense, these taxes would constitute a traditional property tax (Salm 2016); however, they are distinct due to the different types of property ownership (Keilbach and Nann 2010). Analysis of the various taxes indicates that the majority are paid at the transaction stage, meaning they are nonrecurrent taxes for the purpose of revenue (Salm 2016). Furthermore, exclusion of owner-occupied residential properties from recurrent property taxes (including both the House Property Tax and the Urban and Township Land Use Tax) by the central or the local authorities constrains local revenue generation. Therefore, despite a property tax on the ownership of private residential properties being a potential source of sustainable municipal revenue, China remains one of a select few countries globally to not employ such a tax (Liu 2018). This exclusion has led to significant criticism and the view that gaps in public revenue are the consequence of a tax system that is weak and encumbered and requires reform.

Liu (2018) highlights that China missed an opportunity to implement a property tax before the housing boom, and if it is not implemented, finding a sustainable own-source of municipal finance will continue to challenge municipalities. In view of the myriad of problems relating to, *inter alia*, economic inefficiency, inequitability, and cost, a preponderance of literature invariably recommends a pertinent need to reform the Chinese property tax system in favor of market-value-based taxation (Gao 2005; Jia and Zhuo 2006; Tao 2006; Bird and Slack 2004; Salm 2016), with Hou, Ren, and Zhang (2014) proposing a design of the property tax system for China. Moreover, a burgeoning corpus of literature argues that Western taxation models should be adopted (Zhang 2003a, b; Xing 2004; Sun and He, 2006), while other reform proposals highlight measures such as combining taxes, reducing tax rates, adopting uniform tax rates, and strengthening property tax legislation and administration (Mao 2005; Xiao 2005; Dong 2006; Ng 2006). Despite the clear need to implement institutional restructuring of the Chinese taxation system, progress and reform have been piecemeal and slow. Indeed, although the implementation of a market-value-based property taxation system was contemplated at the third plenary session of the 16th Chinese Communist Party Congress (October 2003), there has been relatively little progress. That said, more recently, the 19th National Congress of the Communist Party of China, held in October 2017, emphasized ongoing fiscal policy reform, through the taxation system and improvement of the local tax system. Pertinently, for the first time, the government proposed principles governing the diffusion of a property tax on the ownership of private residential properties (Liu 2018).

Recent Developments in Property Tax Reform

The Chinese central government has been exploring the possibility of reforming its current land and property tax system since 2003, while at the same time putting an end to excessive taxes and fees on real estate development and transactions. Such reforms aim to generate significant revenue for local governments by establishing a system to tax existing property premised on assessed value, on an annual basis. Since then, considerable progress has been made in establishing land and property registries in Chinese cities. In 2010, the State Administration of Taxation (SAT) ordered that every province must choose at least one city to experiment with property value assessment in order to verify the housing sale price self-reported by home purchasers for the deed tax. Similarly, China has not been slow in developing appraisal technology. Indeed, following three decades of development, considerable headway has also been made in developing computer-assisted mass appraisal (CAMA) technology that matches the Chinese urban setting. A mass appraisal system was implemented in Guangzhou in 2007, providing the government with an objective and equitable property value database and an important tool for informing policy decisions on market regulation and facilitating market transparency with reference prices for all market participants (Liu, Sun, and Han 2006).

"In 2005, the SAT compiled a Real Property Assessment Valuation Regulation Trial that specified 12 chapters and 40 provisions covering data collection, standards, and the CAMA system. All the pilot cities have finished the simulation assessment and calculated the tax burden and tax revenue according to different tax rate scenarios. In 2011, at least one city in each province had been selected to conduct property value assessment of newly purchased property for the collection of the deed tax, and in early 2011, Shanghai started to collect taxes on newly purchased second homes of residents and first homes of nonresidents based on transaction value, representing an important milestone for tax reform" (Man, 2012:18).

Most recently, in 2016, an Appraisal Law was promulgated in order to establish the legal status of the appraisal industry.

A further step in the nationwide introduction of the property tax was undertaken in 2014 with the establishment of the Bureau of Real Estate Registration, a valuation-pushed approach with little attention paid to the taxpayer service, collection, and enforcement side. The focus was initially on property registration, with the bureau subsequently responsible for drafting and enforcing land management regulations and resolving land disputes. Currently, the Standing Committee of the National People's Congress, China's top legislature, is working on property tax legislation. The legislation will most likely include taxation on both housing and land: a housing tax on homeowners and a land tax on land developers.

The 19th National Congress of the Communist Party of China, held in October 2017, continued to emphasize fiscal policy reform. Specifically, it called for the widening of taxation system reform and the improvement of local tax system (Liu 2018) by increasing the share of direct taxes. For the first time, the government recently unveiled three principles for the rollout of a property tax on the ownership of private residential properties: the passing of a property law by the National People's Congress, central government authorization to local governments for implementation of property tax, and step-by-step rollout premised on property value assessment (Liu 2018).

Challenges for Property Tax Reform

Although the evolving nature of fiscal reform is positive, tax-based revenue generation remains limited, and many subnational governments in China continue to experience fiscal stress and incur major local debt because of the large fiscal gap between expenditure responsibilities and revenue capacity — unfunded mandates (Duda, Zhang, and Dong 2005; Man 2011; Salm 2016). Furthermore, although the literature has pointed to pilot studies of market-value-based property taxation, little attention has been paid to institutional constraints and the need for wholesale technological and administrative restructuring, transformation, and capacity-building. These are needed to allow the collation of reliable and validated data and transparent property appraisal practices supported by, *inter alia*, effective CAMA technology; collection and enforcement mechanisms; impartial, efficient, and low-cost adjudication processes; and training support for staff to operate the system effectively.

Salm (2016) argues that the issues relating to property-related taxes in China can be grouped into a number of core issues. First, since the 1994 tax reform that introduced the Tax Sharing System (TSS), under which specific tax revenues are assigned, there has been a mismatch between local revenue and expenditure assignments. Such a mismatch continues and has been acutely observed in the megacities, where budget expenditures exceed general budget revenues, with property taxes accounting for 5 percent of local tax revenue. In addition, local discretionary powers remain limited, and local governments may vary property tax rates only within a centrally determined range. Salm further argues the Chinese local tax system is largely premised on windfall tax revenues collected at the transaction stage. The size of these nonrecurrent revenues relies primarily on external determinants, and revenue can be volatile because of the quantity and availability of land sold.

The constraints of state-owned land and increasing population densities are limiting sustainable revenue, meaning the windfall tax revenue model is time-limited. A major source of concern is that revenues from the urban property tax are static because of the constrained tax bases and the fact that urban residential properties (the bulk of the potential tax base) are exempt. At the same time, property owners live under a "veil of uncertainty" (Man 2012) because, despite improvements in property rights law, private property might revert to state government in the absence of land use renewal. In a similar vein, pilot programs were not supported with legislation, and this ultimately renders enforcement of property tax arrears difficult, although China's legislative body recently expanded the purview of property law nationally.

Further significant hurdles remain, in the form of political will and public appetite. Liu (2018) highlights that the strongest opposition to the introduction of a property tax on the ownership of private residential properties emanates from the reality that more than 90 percent of urban households own one or more housing units.

He highlights that the central government studied the feasibility of property tax around 2000, and this led to the seminal publications of Xie (2005, 2006), who described this as a huge missed opportunity for the introduction of the tax in 2003, a few years after the housing policy reform and before the start of the housing boom.

In noting the underlying resistance to property tax from homeowners, Xie advances a number of possible solutions for policy makers in the design and implementation of the property tax. In this context, he contends the deficient price structure of public land leasing needs to be reformed by introducing the property tax and a public land rental charge. This structure would also give municipalities some degree of flexibility to use public land rental charges as a policy tool to stabilize land costs.

Liu (2018) further contends that property tax could be introduced immediately, if it were oriented toward a wide tax base at very low rates; this may allow it to gain acceptance yet generate much needed revenue. In the alternative, he also suggests a "grandfathering" approach, which is transitional and gradually ensures all residential property is subject to taxation. A further approach is to allow for a period of transition by delaying the effective implementation of property tax law for a period of time for homeowners to adjust to housing portfolios.

Finally, in recognizing ongoing change, Liu (2018) argues that while the majority of municipalities are not yet ready to implement property tax (because of administrative and assessment deficiencies), some of the large municipalities have made intelligence and system gains and therefore have the ability to proceed with advancing property tax law. Provincial governments with such readiness could be rewarded with incentives or discretional rates, while others could be targeted with development programmes.

Mass Appraisal and Modeling Approaches

Mass appraisal in the Chinese context has been investigated by a number of authors (Yicheng and Chuanrui 2005; Zhou et al. 2018; Jijin 2011; Yongfa and Lei 2009). The findings generally support the introduction of mass appraisal while recognizing the peculiarities of local conditions, some of which may require the adoption of local area standardized values and other approaches to deal with thin markets and allow inexpensive and straightforward updating of values.

This approach was also the basis of the work conducted by Jijin and Yan (2012), which presented the concept of "municipal unity valuation." In order to achieve high precision, low cost, and easy updating in the valuation of municipal real estate, they constructed a unity valuation model for Shenzhen city, and the analysis exhibited the approach to be applicable and pragmatic. In a similar vein, Yiping (2007) suggested an approach based on benchmark property values. Pertinently, the literature illustrated that moving forward a value base is achievable, albeit with caveats regarding the extent to which full discrete market valuation of every property is feasible.

There is also an emerging body of research that has begun to "'push the boundaries" of mass appraisal practice by investigating the incorporation of geographic information system (GIS) methodologies to enhance the accuracy of real estate data. Indeed, Geng and Li (2011) establish a mass assessment model combined with GIS to use long-term trend and cost methods. Similarly, Liu, Li, and Wang (2015) examine the requirement for using GIS spatial analysis methods coupled with a VIKOR (VlseKriterijumska Optimizcija I Kaompromisno Resenje; multi-criteria optimization and compromise Solution) methodology for enhancing real estate mass appraisal. With their "system," they calculated predicted real estate prices, revealing high accuracy with actual prices, and concluded that their approach can provide technical support for the levy of estate duty in mass appraisal systems.

Indeed, China has somewhat paradoxically led the way with GIS-based three-dimensional (3D) mass appraisal modelling and, for the Shenzhen region, arguably has developed the most prominent 3D valuation-based system globally, in joint collaboration between the Shenzhen Centre and ESRI Canada (a property tax on the ownership of private residential properties). This is the subject a research study by Zhang et al. (2014), which applies GIS 3D modelling and analysis technology. Integrating a procedural modeling approach and two-dimensional (2D) GIS data of Shenzhen, the research generates 3D external models of buildings and a 3D internal model, using vectorization of the property distribution within the target building.

Using GIS visibility analysis to account for the landscape and sunlight, the authors are able to establish concrete quantization indexes, such as landscape visual range and sunshine duration, which is weighted to synthesize a valuation. Zhang et al. (2014) view this more precise 3D visualization effect to provide appraisers with a more intuitive and efficient view for real estate appraisal and to greatly improve the efficiency and accuracy of real estate appraisal, a process described as GAMA and developed by ESRI Canada (for a full discussion, see Nunlist 2017).

Moreover, this system is being utilized to better execute property transaction taxes whereby only 27,106 challenges have been made (effective January 2017) with only 282 assessments needing readjustment based on millions of properties valued (Nunlist 2017). Despite these heralded successes, the Shenzhen assessment project is not without its challenges. Primarily because of market infancy, there is a relative dearth of transaction data and notable instances of under reporting of price to mitigate transaction taxes (Nunlist 2017).

Numerous studies have developed an understanding of the theoretical underpinnings and the viability of the rollout of mass appraisal in China. The literature shows a wealth of research methodologies that can be and have been adopted to assess the challenging nature of the Chinese urban form. The drive toward implementing mass appraisal has shown promise; a number of research papers have been able to do this at the city/regional level; and importantly, a number have specified the models using "real" property transactions, which was traditionally a limitation (Yicheng and Chuanrui 2005). These developments have undoubtedly been helped by the increasing availability of reliable price data from more robust property companies and online multiple listing sites.

A core challenge is also related to the implementation (and effects) of property tax reform. While studies have emerged, they are limited to specific regions or cities, resulting in limited insights into the effect on households and significantly on potential "winners and losers" in light of any reform.

The heterogeneity of the market also presents some unique policy challenges to any blanket approach. This can, however, be overcome by introducing more spatial approaches coupled with integrated technological solutions and data, which are also emerging in parallel, offering the opportunity to blend efficiency and equity within modelling frameworks, which can be tailored on a region-by-region basis.

Moreover, while a plethora of studies have investigated the potential of using the mass appraisal approach — and the usefulness of integrating GIS to enhance data provision for explainability — there is an outstanding requirement for assessing whether mass appraisal models developed for China would conform to international benchmarks. Data continue to be a challenge, namely, their availability and robustness for wider application, given that existing studies are regional or city specific. While there has been extensive refining of sophisticated methodologies for appraisal purposes, these methodologies remain limited to a few regions and therefore are somewhat idiosyncratic. There remains a gap in terms of a nationwide examination and feasibility analysis for CAMA scalability, given an assumption of a policy requirement for ad valorem property tax reform. It is this gap that this research seeks to bridge.

Data

Nationwide transaction data in China are relatively thin and difficult to obtain from official sources, a situation further exacerbated by the extreme variation in robust sources and reporting mechanisms. Similar to Mou et al. (2017), the national real estate transaction data used in this paper were compiled from a number of websites (http://www.fang.com/; https://www.anjuke.com/; https://www.lianjia.com). These websites are operated by large property companies that provide data for thousands of branches in Tier 1 to Tier 4 cities in mainland China.

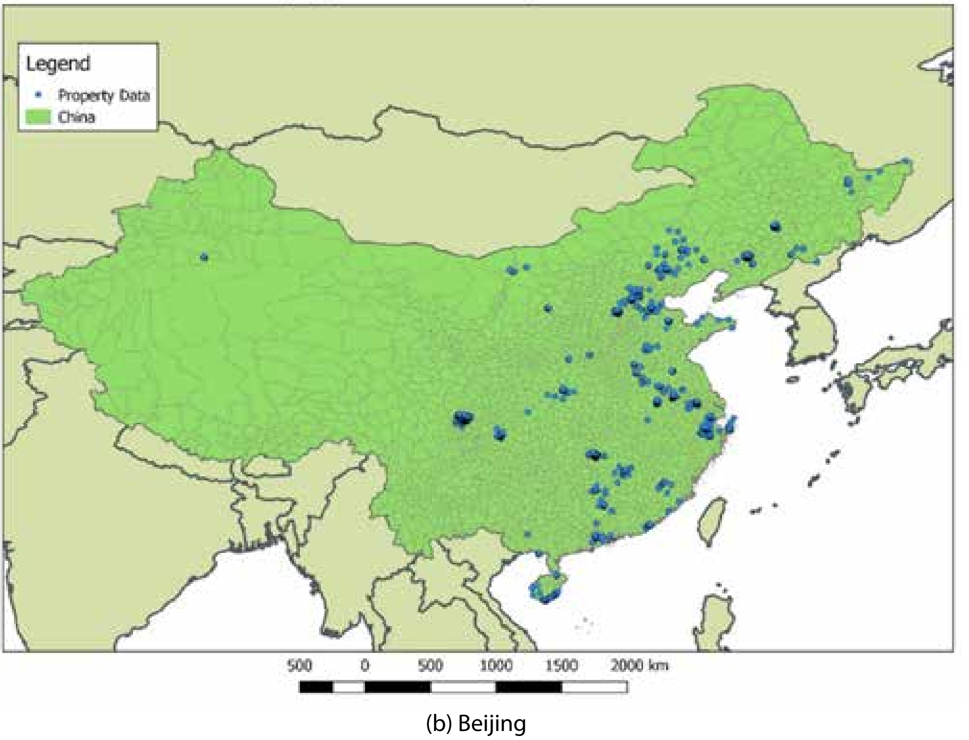
For example, initial inspection of one of the sites provides coverage for 658 cities (Tier 1 to Tier 4) and 29 regions, each of which allows dissection at both the subdistrict and subcity prefectures level. (As a consequence of these websites providing only listing pricing information, we endeavored to validate the analysis for a random subset of markets using sales data from other sources in order to test for potential discrepancies and adjust accordingly.)

The research used a web-crawling exercise with the data acquired through various private listing companies by programmed web scraper and crawling methods (only when legally and ethically permissible, and not in violation of any terms of service) across the various listings, acquiring an initial extraction of 26,579 records and further supplemented with 46,857 records for the Beijing and surrounding hinterland market region, providing 73,436 observations in total (see figures 1a and b).

The information contained within the listing price data is rich and encompasses a wealth of structural and neighborhood characteristics typically used for modelling purposes. An initial exploratory investigation of the variables is shown in table 1.

Figure 1. Spatial representation of the data observations

(a) National overview



(b) Beijing

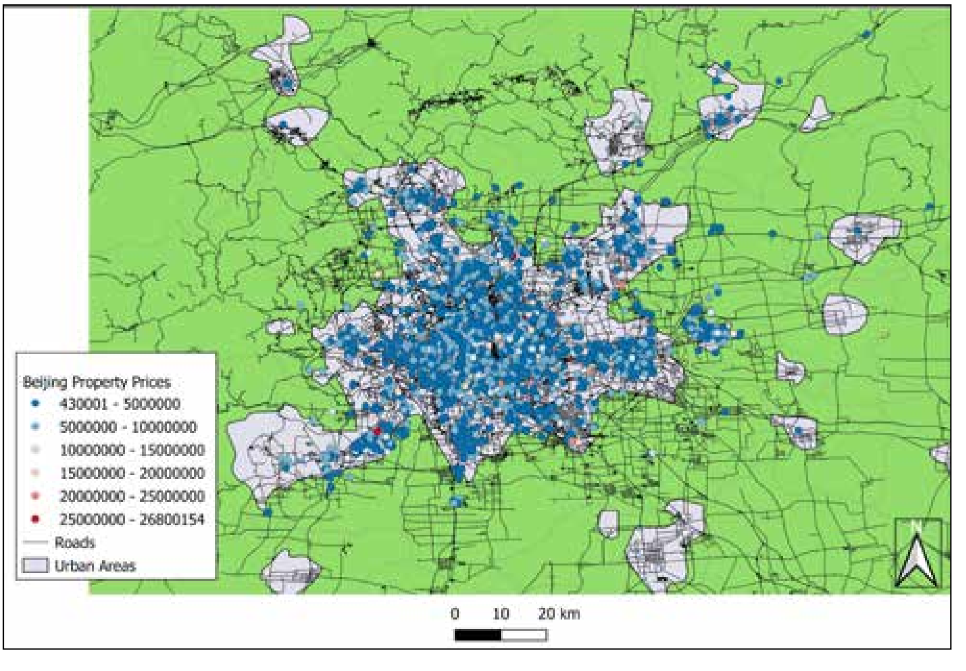


Table 1. Exploratory variables and descriptions

Variable	Description
Floor area	Size of property, square meters
Average price per square meter	Price of property per square meter
General condition	Categorical specification based on assessment
View/aspect	View and orientation /direction (facing) of the property
Floor level	Categorical (high, middle, low) of level in building block
Location	Address, X,Y of property
Build year	Year of build of property
Build type	Public or private market build
Elevator	If property (block) has an elevator
Transaction volume	Level of transactions (monthly) in a district (percentage change)
District price	Average price of property per district
Nearest subway	Distance to nearest subway (meters)
Commodity housing estates	Defined neighborhood/submarket areas for developments

The data were scrutinized for data entry error, missing observations, and non-normal or nonstandardized properties offering the potential to have an impact upon model functionality and reliability. Initial frequency analysis shows a wide variation and range across a number of property characteristics. Overall, this data-cleansing exercise highlighted that a number of these erroneous data entries and problematic observations were cross-correlated, thus removing 180 observations in total. Having purged erroneous and missing entries from the data, we used initial diagnostic analysis to identify outliers within the sample data. There are a number of statistical-based approaches for removing outliers. This research tested a combination of Cook's distance and Mahalanobis distance to estimate the level of the (undue) influence of a data points with large residuals (outliers) and/or high leverage, which may distort the outcome and accuracy in regression-based analysis.

Mahalanobis distance is a multidimensional generalization of the idea of measuring how many standard deviations away P is from the mean of the distribution (D). This distance is zero if P is at the mean of D and grows as P moves away from the mean along each principal component axis. If each of these axes is rescaled to have unit variance, then the distance corresponds to standard Euclidean distance in the transformed space. The distance is thus unitless and scale-invariant and takes into account the correlations of the data set.

At the aggregate level, encompassing all data, this initial inspection illustrated extreme instances of outliers; however, when disaggregated by geographic location (city level), initial model diagnostics display relative normality in the residuals. (For example, the city of Baoding had 14 cases of standardized residuals beyond the acceptable threshold; however, this is out of 1,239 observations, or 1.12 percent.) The suite of property and locational characteristics was subsequently transformed into a binary state (where applicable) to permit various modelling specifications and procedures to be tested (table 2).

Table 2. Included variables and transformations

Attribute	Description	Transformation
Property age	Age of property (years)	Binary (1 if 39 years old; 0 otherwise).
Orientation	Orientation of property	Binary (1 if east; 0 otherwise)
Specification	Condition and finish of property	Binary (1 if luxury-end; 0 otherwise)
Floor level	The level at which the property is located	Binary (1 if high; 0 otherwise)
Total floors	Total number of floors in the building	Scale
Bedrooms	Number of bedrooms	Bedrooms (1 if bed 1; 0 otherwise)
Bathrooms	Number of bathrooms	Bathrooms (1 if baths 1; 0 otherwise)
Area	Size of property (square meters)	Scale
Property type	Type of property	Binary (1 if high-end apartment; 0 otherwise)
City[a,b]	Location city of property	Binary (1 if Baoding; 0 otherwise)
Administrative district[a,b]	Administrative district in which a property is located	Binary (1 if Baoding; 0 otherwise)
Submarket[a,b]	Submarket area in which a property is located	Binary (1 if Baoding; 0 otherwise)

Methodology

For automated valuation models (AVMs) to produce accurate, uniform, and defensible values, the completeness and reliability of data are of fundamental concern. In regard to a policy discussion approach and the realistic consideration of an ad valorem property tax enactment in China, analysis of whether data are adequate and capable of yielding valuations, in line with internationally accepted property tax standards, is needed (Deng 2005). Consequently, utilizing property transactions from varying residential markets, this research evaluates a nationwide assessment of the feasibility of mass appraisal valuation for the nation of China. A comparison of model performance extends beyond typical regression diagnostics toward IAAO standard metrics, which appraise valuation fairness, equity, and uniformity. This research therefore determines the feasibility of creating AVMs in China capable of conforming to IAAO valuation standards, with implications for scalability across national markets. In addition to increasing the understanding of real estate markets and appropriate property tax AVM methodologies in China, this research can guide the adoption of valuation policy prescriptions for economies with similar markets and/or similar data to China.

Measuring Non-uniformity and Inequity in Value-Based Property Taxation

IAAO develops and maintains statistical standards by which governments measure, track, and compare valuations with respect to various performance measures, including assessment uniformity and equity (IAAO 2013). Such statistical analyses are referred to in the property tax arena as ratio studies. The coefficient of dispersion (COD) measures the uniformity of an assessment stratum and provides a measure of the variation of the individual assessment-to-sale (ASR) ratio around the median ASR. (The ASR is a common way to measure valuation accuracy within property tax valuation, in which the estimated value of a property is divided by the sale price, or in regression terminology, the predicted value, $\hat{y}$, is divided by the observed value, y; valuations with an ASR greater than [less than] one are considered overvalued [undervalued]). If the individual ASRs are clustered closely around the median, the COD is low, which suggests the assessments are relatively uniform. However, if the individual ASRs vary widely from the median, the COD is high, which indicates that the property was not uniformly assessed. Statistically, the COD expresses the average absolute deviation of the individual ASRs from the median ASR as a percentage of that median. It is represented by the following formula:

$$COD = \frac{100}{R_m}\left[\sum_{1}^{N} \frac{|R_i - R_m|}{n}\right] \tag{1}$$

where

R_m = median ASR
R_i = observed ASR of the i-th sale
n = number of properties sampled.

The price-related differential (PRD) is used to indicate assessment uniformity and to quantify the degree of regressivity, in which the low-value properties are over-assessed relative to the high-value properties, or progressivity, in which the low-value properties are under-assessed relative to the high-value properties. It is calculated as follows:

$$PRD = \frac{\sum_i \frac{\left[\frac{\hat{Y}_i}{Y_i}\right]}{n}}{\sum_i \left[Y_i * \left(\frac{\hat{Y}_i}{Y_i}\right)\right] / \sum_i Y_i} \tag{2}$$

where

$\hat{Y}_i$ = predicted sale price of the i-th sale
Y_i = observed sale price of the i-th sale
n = number of properties sampled.

The benchmark range for the PRD is 0.98 to 1.03 (IAAO 2013). If there is a tendency for the ASRs of high-value properties to be lower than those of low-value properties, the PRD is greater than 1.03. If, on the other hand, high-value properties have higher ASRs than low-valued properties, the PRD is less than 0.98. In this regard, the PRD measures the pattern of inequity in assessments that has a correlation with the value of the property. This pattern has important policy implications as appropriate vertical equity measurements can indicate whether relative tax valuations are fair and equitable, or whether an undue burden is falling on poorer households that have a lower ability to pay the property taxes.

Valuation Methodology

There are a variety of spatial based modelling frameworks in existence for examining house prices and undertaking valuation practices, principally AVMs. This study employs both a traditional ordinary least squares (OLS) and a more spatially local weighted regression methodological approach, namely, geographically weighted regression (GWR).

OLS (Spatial Regime) Model

The basic objective of multiple regression analysis is to develop a strong predictive relationship between property characteristics and value, so that the latter can be estimated through knowledge of the former. The semilog linear fit is applied within the modelling frameworks because of computational efficiency and interpretability, which provide useful interpretations of the independent variable coefficients in terms of their elasticity in respect to the dependent variable. The semilog specification is as follows:

$$LnY = \beta_0 + \beta_1.X_1 + \beta_2.X_2.......\beta_n.X_n + \varepsilon \tag{3}$$

where

LnY = the dependent variable (log of sale price)
$X_1......X_n$ = the independent variables
$B_0 ....\beta_n$ = parameters to be estimated
ε = the error term.

Geographically Weighted Regression Model

Locally weighted regression (LWR) is an extension of traditional OLS that has demonstrated, in certain cases, superior performance in explanatory power (Brundson, Fotheringham, and Charlton 1996; McMillen 2010; Brundson 1998). GWR is an LWR technique that allows observations in closer geographic proximity to a subject property to receive more consideration than those further away (Fotheringham, Brundson, and Charlton 2002). Real estate markets characteristically behave differently over geographic space, and price determinants may vary greatly by location. Conventional OLS models are often unable to accurately account for spatial variation, resulting in a spatial correlation of error terms (spatial autocorrelation) with dummy variables used to capture and isolate locational variations not fully correcting for spatial autocorrelation (Fotheringham, Brundson, and Charlton 2002; McMillen 2010). With regard to IAAO ratio study standards, evaluation estimates produced by GWR mass appraisal models have been shown to achieve superior results in comparison to OLS models (Borst and McCluskey 2008; Moore and Myers 2010; Lockwood and Rossini 2011; McCluskey et al. 2013; Bidanset and Lombard 2014a). GWR is represented by the following formula:

$$yi = \beta O(xi,yi) + \sum \beta k\,(xi,yi)xik + \varepsilon i \tag{4}$$

where

y_i = i-th sale
β_0 = model intercept
β_k = k-th coefficient
x_{ik} = k-th variable for the i-th sale
ε_i = error term of the i-th sale
(x_i, y_i) = x, y coordinates of the i-th regression point.

The approach allows coefficients to vary continuously over the study area, and a set of coefficients can be estimated at any location, typically on a grid so that a coefficient surface can be visualized and interrogated for relationship heterogeneity. GWR makes a point-wise calibration concerning a “bump of influence” around each regression point whereas nearer observations have more influence in estimating the local set of coefficients than observations farther away (Fotheringham et al. 1998). In essence, GWR measures the inherent relationships around each regression point, i, where each set of regression coefficients is estimated by weighted least squares. Within this study, the weighting scheme W_{i} is calculated with a kernel function based on the proximities between regression point i and the N data points nearby. A number of kernel functions can be used for the weighting scheme, a plethora of kernel densities that can be implemented and that can have various impacts upon ratio study performance (see Gollini et al. [2013] and Bidanset and Lombard [2014b] for a full discussion). In GWR, an nXn spatial weights matrix is constructed to indicate the weight applied to each observation, assigned relative to the subject based on geographic distance:

$$w_{ij} = exp[-d_{ij}/b^2] \tag{5}$$

where

w_{ij} = weight applied to the j-th property at regression point i
d_{ij} = geographical distance in kilometers between regression point i and property j
b = geographical bandwidth.

The bandwidth in GWR specifies the radius of the weighting function. It is fixed, based on absolute distance, or adaptive (fluctuating), based on a predetermined number of nearest neighbors. An optimized bandwidth may be identified based on various conditions, but is most commonly that which corresponds to a minimized cross-validation (CV) or Akaike information criterion-corrected (AICc) scores (Fotheringham, Brundson, and Charlton 2002). This kernel specifies how weights are calculated and assigned to the observations with the kernel implemented shown to hare an impact on ratio study performance (Bidanset and Lombard 2014b, 2016; Bidanset et al. 2017a, b). This study utilizes the Akaike information criterion (AIC) (Akaike 1973), which accounts for model parsimony (i.e., a trade-off between prediction accuracy and complexity). This research subsequently compares the ratio study performance of each kernel approach across provinces in China, identifying mass appraisal modeling approaches that optimize property tax fairness and equity. The spatial function bi-squared, adaptive kernel using 10–15 percent neighbors is employed with an optimization using the golden section search and the AICc.

Model Selection and Multimodel Inference

To ensure model parsimony and appropriate model selection, initial testing of parameter selection was undertaken to reduce model complexity without reducing model predictability. This model selection procedure, based on minimizing the AICc, ensures retention of the highest level of explanation as depicted by the adjusted R^2, is undertaken to reduce the model form, examine the most parsimonious spatial model, and remove unwanted influential variables and multicollinearity. Within this research, this process is based on 12 variables selected culminating in 2,047 models tested. The results of the selection procedure filtered by the AICc revealed that the most parsimonious model form excluded the restricted (embargoed) sale variable and higher community parameter estimates for the best OLS model, indicating that they do not add value in terms of importance and significance.

Empirical Results

A series of log-linear models are developed to investigate the nature of the deterministic effect of the structural and locational characteristics on property price. The models are developed systematically in order to establish the various levels of significance attributable to the property characteristics and varying levels of spatial geography. A noticeable and interesting finding is that there appears to be a challenge in terms of the property size (area)-price relationship that commands a relatively low level of explanation in comparison with other traditional real estate markets. The base OLS models (excluding any spatial representation) constituting property structural characteristics show 37.7 percent of explanation, again a finding that is generally lower than expected (table 3). When each respective city is factored into the model architecture, the model shows significant improvement in terms of explanation (77.6 percent). Introducing further spatial dummy variables based on administrative areas and at the more granular level, submarkets, the explanation increases to 87.6 percent, providing an excellent basis for undertaking mass appraisal exercises.

Table 3. OLS and GWR logarithmic model summary

Model[a]	F-statistic[b]	R^2	Adjusted R^2
OLS (city + admin + sub)	176.271***	0.881	0.876
OLS (city +admin)	475.243***	0.828	0.826
OLS (city)	913.559***	0.776	0.776
OLS (base)	262.755***	0.378	0.377
GWR	-	0.823	0.811

[a] *Dependent variable: LnPrice.*

[b] ****denotes 99% significance. Full model coefficients available upon request.*

Prediction Accuracy: PRD and COD Ratio Performance Measures

Initial investigation examining IAAO benchmarks, namely, the PRD and COD, for the overall models is shown in table 4. As expected, the base models neglecting spatial information perform relatively poorly for both ratio measures, signify poor uniformity of appraised values, and depict regressivity, whereby high-value properties are under-assessed relative to low-value properties. This is also evident for the COD statistic, which displays relatively high dispersion of assessed value from the median. When the locational characteristics are factored in, the log (city + admin) model falls nominally outside both ratio standard thresholds. Both fully specified spatial models (city + admin + sub) meet the IAAO accepted thresholds for the PRD, with the log model inside the range widely accepted for the COD.

Table 4. PRD and COD ratios

	PRD	COD	COV[a] (Median)
Log (base)	1.166	0.315	44.4%
Log (city)	1.050	0.178	25.9%
Log (city + admin)	1.035	0.155	22.9%
Log (City + admin + sub)	1.024	0.125	18.4%
GWR	0.971	0.152	24.2%

[a] *COV = coefficient of variation.*

City-Level OLS Models

The data are further dissected by each city region to examine the feasibility of specification of an ad valorem model at this level, under a uniform model archetype. Table 5 displays the overall level of explanation of each city represented in the data sample. The results show a variation in the OLS model performance, exhibiting good model fits with high levels of explanation evident (Changsha, 88 percent; Chengde, 90 percent). Nonetheless, there are instances of poor model explanation (such as Shangqiu, 26 percent; Benxi, 24 percent), which require further investigation for value-determining parameters and omitted variable bias. Accordingly, examination of the ratio benchmarks for uniformity and horizontal and vertical equity show a large number of the city models conform to the accepted thresholds for both the OLS and in-sample (training) data performance ratio measurement. These results indicate that there is, on first viewing, a valid basis for developing mass appraisal systems across China's urban housing markets, as evidenced by the high levels of model explanation and conformance with the ratio standards across both the global GWR (training) and OLS modelling frameworks.

Table 5. City-level prediction accuracy

	OLS				GWR Training		GWR Testinga	
City	Adj. R^2	Obs.	PRD	COD	PRD	COD	PRD	COD
Baishan	0.774	580	1.029	13.9	0.9672	7.41	0.436	19.65
Baoding	0.356	1239	1.041	14.6	1.034	11.15	1.177	15.16
Bayinnaoer	0.809	299	1.011	7.8	0.9995	2.6	0.657	12.77
Bengbu	0.877	97	1.004	3.8	0.9689	3.87	1.063	18.19
Benxi	0.241	109	1.014	8.7	0.9045	2.65	0.94	9.934
Binzhou	0.681	116	1.024	11.5	0.9962	3.91	0.777	15.88
Bozhou	0.556	397	1.029	13	0.9867	3.94	1.238	20.60
Cangzhou	0.772	2430	1.023	11.7	0.9946	2.01	1.017	8.052
Changchun	0.582	234	1.009	7.4	1.0363	3.2	1.043	27.99
Changsha	0.880	1580	1.03	14.3	0.9856	4.31	1.498	10.64
Changzhou	0.548	1422	1.016	8.8	0.9975	2.26	0.974	31.98
Chengde	0.904	1929	1.02	11.9	0.9967	2.99	1.059	16.26
Chengdu	0.831	2072	1.015	10.2	0.9937	3.36	0.662	16.35
Chenzhou	0.827	1605	1.02	13.9	0.9976	3.1	1.016	7.781
Chifeng	0.779	838	1.005	4.7	1.0006	1.72	1.167	6.209
Chongqing	0.800	246	1.004	4.7	1.0021	2.51	0.683	18.41
Chuzhou	0.524	1010	1.029	13.8	0.9756	4.97	0.862	6.327
Gian	0.485	461	1.001	2.7	0.9996	0.9	1.586	8.455
Hefei	0.691	544	1.003	3.5	1.0431	2.17	0.511	24.87
Jiamusi	0.741	325	1.029	13.7	0.968	4.86	0.947	12.06
Jiangmen	0.530	466	1.005	5.2	1.0266	2.02	0.951	9.956
jiaxing	0.261	111	1.02	10.5	0.9878	2.17		
Jining	0.823	61	1.006	5.4	1.0246	4.13	-	-
Jiyuan	0.739	105	1.005	5.4	1.0026	2.1	-	-
Sanming	0.685	429	1.023	11.7	0.9985	4.28	1.053	11.744

Table 5. City-level prediction accuracy (cont.)

City	OLS				GWR Training		GWR Testinga	
	Adj. R^2	Obs.	PRD	COD	PRD	COD	PRD	COD
Sanya	0.849	572	1.03	14.7	1.0519	3.43	0.875	22.358
Shangqiu	0.313	202	1.019	9.3	1.0023	2.1	0.796	12.847
Shantou	0.261	649	1.013	8.2	0.9996	1.71	0.999	7.2
Shaoguan	0.708	624	1.01	7.2	0.9985	2.3	0.805	8.416
Shaoxing	0.449	1075	1.047	16.1	0.9957	4.44	1.063	12.63
Shenyang	0.784	1212	1.032	14.2	1.0031	5.22	0.888	23.938
Shijiazhuang	0.890	1373	1.012	8.3	0.9922	2.65	1.552	20.858
Shiyan	0.739	992	1.025	12.5	0.9984	3.53	0.657	13.192
Suqian	0.459	61	1.013	8.6	1.0083	3.73	-	-
The north sea	0.427	100	1.02	10	0.8472	2.66	-	-
The rising sun	0.854	437	1.027	12.6	0.9861	4.66	0.523	21.668
Yulin.	0.304	194	1.043	16.4	0.9975	4.5	0.682	10.166
Zhoushan	0.924	150	1.009	5.7	1.0017	5.13	-	-

When specifying the predictability of the GWR testing (out-of-sample) mode, there appears to be some considerable movement in terms of the ratio benchmark performance, some of which becomes very poor and changes from being marginally regressive to acutely progressive (and vice versa). This behavior is arguably reflective of the underpinning spatial structure of some of the markets in these cities. Indeed, two issues that might appear to have an impact upon this behavior are the floor area-price basis, which results in some areas with low explanation. Within some cities, basic market assumptions for in-sample versus out-of-sample testing do not conform, in light of discontinuities in the urban form as a consequence of structural characteristics, such as community estates that act to demarcate and regulate spatial continuity and thus price differentials based on implicit and explicit pricing with noncontinuous spatial patterns. In essence, the complex mosaic of spatial concentration as a by-product of market characteristics cannot be explained adequately by a holdout sample. As a result, more on-the-ground market contextualization is required for some cities upon initial inspection of the data to complement the introduction of a mass appraisal approach.

Testing the Scalability of Modelling Frameworks

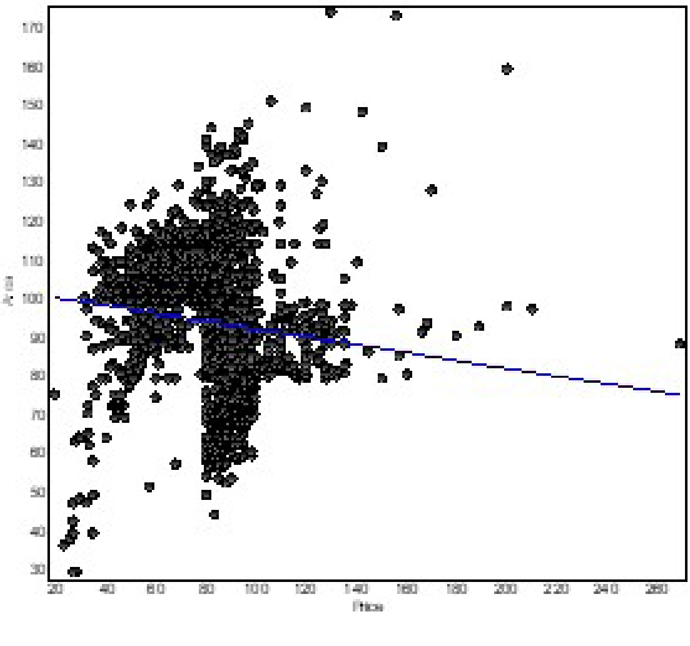
At the global level, the ratio benchmarks suggest that a large number of the cities conform to the accepted thresholds. Nonetheless, this would not be practical or feasible in terms of a mass appraisal approach, and more micro-level analysis is required for implementation. Therefore we select a random assortment of five (spatially dissimilar) cities — Baoding, 1,239; Bayinnaoer, 299; Bengbu, 97; Chongqing, 246; Chuzhou, 1,010 — in order to test a more regional (or subset) model to examine the level of performance and the spatial variation (for differing sample sizes) of the included characteristics, such as property size (area) coefficients, in a more municipal setting. This approach forms the basis of estimation, from a CAMA perspective, of spatial heterogeneity and determination of value-significant attributes for predictive estimation of the sold versus unsold stock.

Examination of the price-area relationship provides an insight into the differing trends across each distinct region. It is clear that there are marginal or partial differences in the fundamental floor area and price association spatially, as observed in figure 2, giving rise to homoscedastic and heteroskedastic bias in the adoption of a wider model at this level. Nonetheless, employing a reduced model structure to account for significant predictors across all city jurisdictions, a GWR model shows an R^2 of 76.3 percent ($F = 109.182$, $p < 0.001$).

The results show a differing pricing effect across the coefficient ranges, signifying a marginal effect, in a spatial sense, with floor area commanding a pricing effect of 0.0048 at the lower quartile to 0.011 at the upper quartile. In terms of model inequity and uniformity, the PRD statistic equates to 1.19 with the COD equal to 26 percent, signalling elevated levels of regressivity and dispersion outside the accepted ratio benchmarks for inequity and uniformity conformance.

As exhibited in figure 2, the predictive accuracy shows structural breaks and, to a large extent, homogeneity of variance with various tangential ”column” structures evident in the data. Also, aspects of heteroskedasticity indicate inflated variances that present questions on model structure and call for further investigation of techniques for increasing robustness, such as boosted regression trees and LASSO/ridge regression, a random forest regression algorithm.

Figure 2. Price–area relationship and predictive accuracy of regional level analysis



The findings seemingly imply that more regionally based CAMA models would be presented with a number of core challenges, even with the application of a consistent parameter subset. The varying, and almost chaotic, nature of the price-floor area (size) relationship — compounded by spatial (dis)aggregation — would seemingly make the introduction of a mass appraisal system, at this scale, demanding if not infeasible. In stating that, international practice would, in any case, suggest that customized models be developed for each district and/or metropolitan area.

Administrative City-Level Models

The analysis is extended to examine the feasibility of city-based mass appraisal models within particular locales. Undertaking this exercise demonstrates the application of the data subset accounting for differing sample sizes and the viability of the data. This analysis takes into account three cities (Chuzhou, Baoding, and Beijing) in order to rationalize the feasibility of the development of city-based valuation models for spatial and market differentiation. Chuzhou is a prefecture-level city in eastern Anhui Province, comprising two Administrative Divisions, two districts, four counties, and two county-level cities. The listed price evidence for Chuzhou city area captures data from seven out of the eight administrative divisions at the administrative district and submarket level. Similarly, Baoding is a prefecture-level city in central Hebei province, approximately 150 kilometers southwest of Beijing, and constitutes three urban districts and two counties. Baoding is ranked seventh among 13 Chinese cities with a population of more than 10 million. Finally, Beijing is analyzed because of its stature as the capital city and because it is governed as a municipality under the direct administration of a central government with 16 urban, suburban, and rural districts.

In terms of market characteristics, in the Chuzhou market three-bedroom properties represent 93.4 percent of the list market, and two-bedroom apartments with two bathrooms for 95.7 percent. Of the 98.8 percent of apartments, specification of no decoration represents 99.3 percent and south orientation (this coefficient encompasses the range from southeast to southwest), 98.5 percent. In the Baoding market, apartments represent 99.4 percent of list prices and two-bedrooms, 83.5 percent. More than 95 percent of the sample data reveals properties are high-end specification. The data illustrate high homogeneity in the housing stock and therefore on initial inspection portray feasibility for mass appraisal exercises.

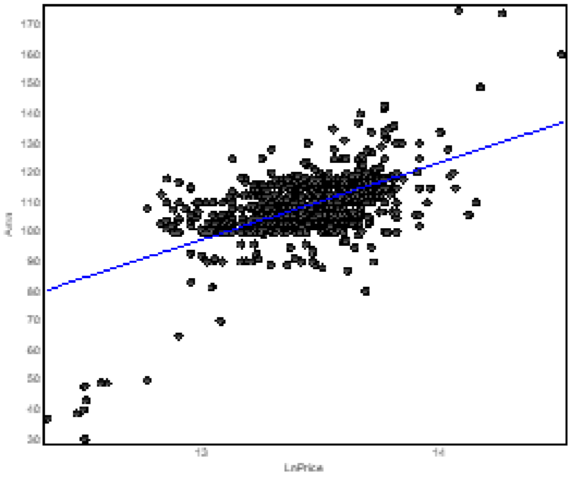
The data for Beijing are relatively rich, in modelling terms, and demonstrate a consistent spatial coverage (albeit in the urban core) as depicted previously in figure 1b. The property stock for the analysis shows more variation and heterogeneity than the other prefectural cities examined. This is perhaps understandable given its long history and arguably more established market system. The data show two-bedroom properties represent 47 percent of the sample, one-bedroom properties 16.7 percent, and three-bedroom 29.3 percent. Apartments dominate at 90.3 percent, duplexes 6.3 percent, and houses the remainder. One-living-room apartments equate to 70.9 percent, and apartments with two living areas 24.5 percent. A similar position exists for the number of bathrooms: one-bathroom apartments account for 71.3 percent and two-bathroom, 24.9 percent.

The price-by-floor-area-relationships for each city are shown in figure 3. For the Chuzhou market the price-size relationship reveals a 34 percent level of explanation, thereby providing a relatively stable basis for determining market value. In contrast, for Baoding, the level of explanation is more characteristic and reflective of inelasticity in terms of price versus size variation. Indeed, this seemingly poses challenges for any floor-area-based tax model and presents a few issues for AVM approaches at the city level.

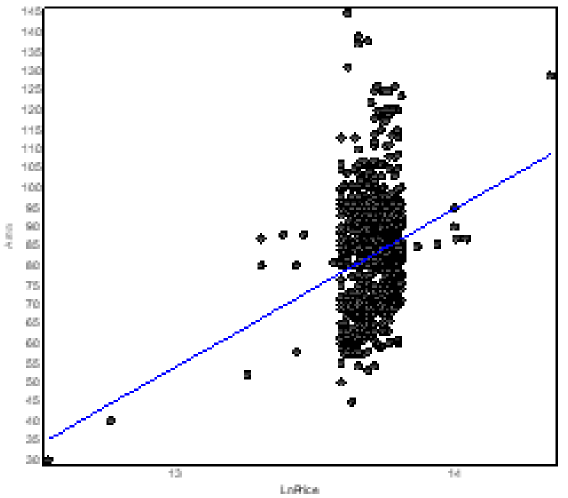
In terms of city model performance, for the Chuzhou region, the model shows an 82.4 percent level of explanation significant at the 1 percent level ($F = 31.06$, $p < 0.001$). The model coefficients show floor area (property size) to be a significant determinant, with both the bedroom and bathroom coefficients conforming to expectation. The results also signal that the floor level the property is situated on is a significant parameter for value, in this instance implying that units located on the lowest floors command more of a premium, a finding also evident for newer properties. The estimates also illustrate different price effects (and significance) across delineated submarket areas. In addition,

Figure 3. Price-area relationship in Chuzhou and Baoding

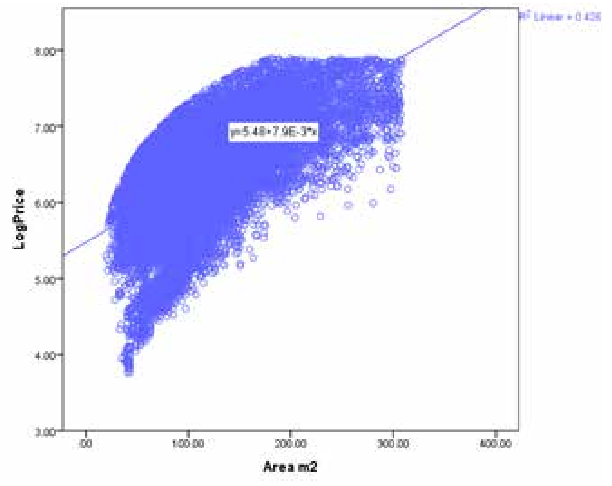
(a)



(b)



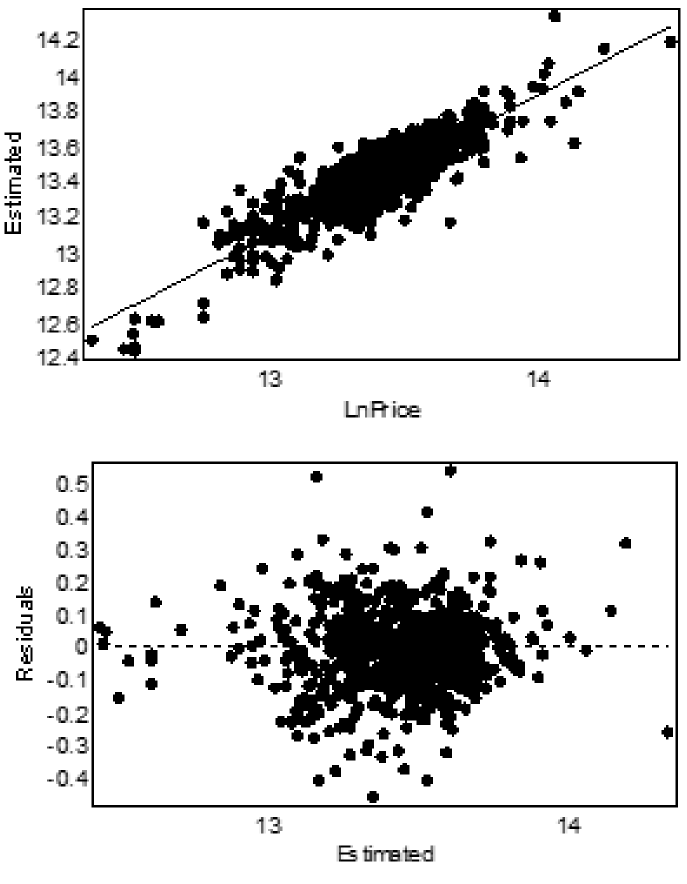
(c)



the GWR model displays an (pseudo) R^2 of 79.1 percent (F = 38.22, p < 0.001). The local R^2 statistics within the GWR model highlight high spatial depiction of the model performance with the R^2 values ranging from 24 to 94 percent and illustrate where the model has more enhanced performance (model coefficients are available upon request} .

The GWR model parameter estimates clearly illustrate the spatial variation in market pricing and the effect of the various property characteristics. The property size coefficient (area) displays an increased effect across the quartiles. This finding is also symptomatic for the bedroom, bathroom, floor range, and age coefficients, which all exhibit both negative and positive influences across the market geography. Accounting for this spatial variation offers important insights and provides a strong spatial basis for isolating market areas where model performance is weak for tailoring more local mass appraisal systems. Indeed, it provides insights regarding further investigation and aspects that need to be accounted for in terms of understanding market substitution, the spatial nature of value significant coefficients, and thus mass appraisal efficacy. In terms of model stability, the standard residual diagnostics (histogram of residuals and the plot of the observed versus predicted values) show the model to be stable with limited error variance and normally distributed residuals (figure 4).

Figure 4. Estimated and actual log-prices and residuals



For Baoding, the model explanation exhibits an R^2 of 35.6 percent ($F = 8.601$, $p < 0.001$). The model-significant parameters reveal a number of archetype physical characteristics that demonstrate statistical significance, such as floor area, age, and low floor level. These parameters provide a platform for more investigative analysis of the implementation of a CAMA approach and clearly demonstrate that identification of additional value-significant attributes (such as "site positive" or "site negative") or further spatial delineation would be required prior to operationalization. For the Beijing market, the overall model performs strongly in terms of predictive accuracy (87.5 percent). Given the increased heterogeneity of the Beijing market, the nature of the property stock variation by type and age, and the relativity of market size, the findings clearly illustrate that market features and characteristics can be readily used and integrated into a mass appraisal system.

In terms of uniformity and equitability standards, the model evaluation for Chuzhou falls within the accepted threshold of the ratio benchmarks, revealing a PRD of 1.029, which displays slight regressivity, and a COD of 13.8 percent. The results suggest good feasibility for introducing ad valorem-based tax assessment in this city area. These findings are complemented by the GWR model in which the PRD equals 1.012 and the COD equals 8.2 percent, providing greater model accuracy and reliability, and, pertinently, demonstrates the viability of introducing a fair and equitable property tax.

For Baoding, the ratio standards are better than expected given the relatively poor level of explanation (35.8 percent). The ratio statistics for Baoding display a PRD of 1.083 and a COD of 12.04 percent, showing it to conform to uniformity and a degree of regressivity. This finding is undoubtedly due to the high degree of homogeneity in the market (sample data) because the data cluster symmetrically. For Beijing, the predictive accuracy of the actual versus the predicted shows a noteworthy deterministic relationship. The PRD (1.01) and COD of 13.1 percent indicate that a mass appraisal model could be adopted and operationalized.

Discussion

The general OLS models show predictive competency from the general set of property and spatial characteristics — a process that is uniform across the cities included. Pertinently, the global mass appraisal performed acceptably for an overall model with spatial dummies, with ratio statistics falling within acceptable IAAO benchmarks. This finding was also evident for the mass GWR-based assessment. However, this was more sporadic when applied to the holdout sample, signifying some preliminary challenges for using the "sold" housing stock characteristics to value the unsold stock in China at this spatial scale. Analysis at the city level highlights increased levels of spatial concentration in geographic clusters, implying price variation at the same location.

These instances of spatial containment appear to result in distinct market structures and thus different pricing levels — certainly vertically (based on the floor level) and marginally horizontally, across the developments — within each designated housing estate. This spatial clustering is compounded by the high homogeneity of the housing stock, which is priced similarly, marginally (partial differences), and differently at all the corresponding locations. This clustering poses challenges for ratio analysis and for investigating issues such as whether there is over-fitting because of the relative homogeneity of the stock and limited variation spatially. This warrants further analysis in terms of integration into mass appraisal modeling.

Indeed, while the city models are not without their "local" challenges, these challenges also afford an opportunity, and the feasibility, to build very robust and standardized models that can readily be adjusted for each region. Nevertheless, a simple pricing on a square-meter basis could present challenges given the reduced level of explanation against price (or floor area) as demonstrated by R^2 values lower than traditionally observed in established market economies. Significantly, deterministic traditional market structures evident in China are not fully in line with those in Western economics, which employ highly specified hedonic models (and spatial models). There do appear to be instances of model over-fitting, which require further investigation for fruitful adoption of mass appraisal exercises, perhaps indicative of rapid, (new) homogenous development.

The level of explanation being achieved does not take into account market tastes or further socioeconomic (market-based) profiling. It may be necessary to apply more basic model formats, initially, to ascertain value-significant market characteristics. Indeed, a consideration for both feasibility and scalability is the concept of "community" or "scheme-" based valuation models — which may be achievable if wider amenities, facilities, and "rights" such as health services are implicit in the pricing of housing estates and developments. More basic model formats could also take the form of more simplified valuation approaches such as calibrating floor area to create a value-weighted floor area and then perhaps undertaking a banding approach.

Conclusions

This analysis has been based on a large data set of list prices of residential property drawn from a number of Tier 1 to Tier 4 cities across China. The data sets utilized provide evidence of the scale and nature of market information available to underpin mass appraisal activity in China. Any operationalized tax policy would almost certainly develop more sophisticated data sets, potentially linked to transacted prices from verified market transactions (ordinarily derived from transfer tax declarations). In addition, any such data set may not adequately represent the full range of properties that would have to be valued in a full ad valorem exercise, perhaps being skewed toward newer properties or possibly overly spatially concentrated, leaving older properties and some locations under-represented.

Nevertheless, the data set does represent a significant data resource that covers many of the major population centers and prefecture-level cities and is therefore a solid "test bed" for investigating both the feasibility and scalability questions central to this research. For the feasibility question, it is evident that it is possible to acquire significant data sets of property prices and associated (potentially) value-significant attributes, which are ostensibly in the public domain and subject to scrutiny. This in itself is a considerable finding, as many emerging economies do not have "live" market data sources to access for this type of research.

After routine data-cleansing and -purging exercises, initial modelling using standard and multiplicative OLS approaches, augmented by GWR techniques, allows us to adequately model the list prices in line with internationally accepted benchmarks of accuracy and uniformity, and increasing performance is achieved because the crucial element of location is more explicitly included within the model specification. To the extent that the data are representative of the population at large, the initial findings suggest that mass appraisal is practically feasible for urban areas in China — adequate market data appear to exist that meet the requirements for hedonic analysis, particularly with a spatial dimension.

In terms of scalability, the results are not uniform across all markets. There are areas with limited data that cannot adequately be modelled, a dimension to be mindful of for mass appraisal exercises. In some areas the models perform slightly less well and may require better data or more tailored calibration. Several models appear to work *too well*, raising concerns about the data and the underlying pricing mechanism, which may be being overtly driven by a more rudimentary or simplistic pricing mechanism in largely new, large-scale, and uniform development areas. This concern may well “unwind” as more properties enter the resale markets and normal market forces begin to take precedence over a ”developer input cost” model of pricing. In addition, there are many ways in which submarket calibration could be carried out, with physical aspects such as the pricing effect of altitude in high-rise developments not being uniform from place to place (perhaps driven by air quality parameters or issues such as perceived fire safety and quality of elevator maintenance, not contained within the data). However, it is highly unlikely that a single model form would be deployed across such a vast socioeconomic and physical geography, so this is entirely acceptable.

One important dimension that has surfaced during the model-building exercises is the issue of market structure, topography, and the value-enhancing attributes. The complex noncontinuous urban form and structure, principally the designation of “community value,” appears to create isolated or noncontinuous pricing relationships, which inhibit the varying nature of price determination. These distinctive market characteristics appear to distort wider scalability of basic tax models for CAMA exercises and to be a barrier for implementation of any spatial framework. Indeed, an inherent problem is that more simplified approaches may not conform to accuracy requirements and, in particular jurisdictions, may face challenges of fundamental market value basics. These basics may include floor area, with more sophisticated approaches simultaneously introducing omitted variable bias and mis-attribution; however, they may be needed for trying to assess extreme locational fluctuations. To account for spatial heterogeneity and model accuracy, a consistent challenge regarding singularity for the spatial weighting matrices emerged. The challenge was found when more enhanced spatial modelling frameworks are being tested, namely, spatial error, spatial lag, and conditional autoregressive models, in addition to GWR.

The main finding is that from place to place, and with few exceptions, available market evidence can facilitate an adequate basic valuation exercise. From a number of typical value significant attributes, robust models can be built that conform with standard horizontal and vertical inequity tests. In this regard it can be argued that there are no fundamental barriers to scalability. The model findings thus demonstrate a good ability to utilize value-significant coefficients in a wider model, with more tailored models also showing promise at the city and administrative levels.

The scale and nature of the data sourced and deployed for this research augur well for efforts to operationalize mass appraisal in China, at least within the market sector. Areas that demonstrate very narrow pricing variation or areas with thinner markets may well benefit from consideration of more simplified approaches, particularly in the billing mechanism, from the perspectives of benefit tax and efficiency of taxation. To implement a nationwide ad valorem property tax, China needs to become more transparent with transacted market data. Setting that debate aside, there is nevertheless adequate evidence that recognizably modern mass appraisal approaches can be devised and deployed to support national coverage of a property tax in China.

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A Gini Measure for Vertical Equity in Property Assessments

BY CARMELA QUINTOS, PH.D., MAI

Abstract

This paper aims to show how the Gini-based measures for inequality, commonly used in the socioeconomic literature, can be applied to property assessments. The index for vertical equity is the ratio of the Gini-based coefficient of assessment to the Gini coefficient of price. It is interpreted as the elasticity of *shares* of assessments to *shares* of prices when prices are *ordered* from the lowest to highest price levels.

A second index is based on the difference, rather than the ratio, of the Gini-based coefficients. An important distinction between both indexes and the price-related differential (PRD) and currently used measures is that Gini-based analyses do not use sales ratios (assessment/price ratios), which are basically the behavior of the appraisal errors. Instead, they are based on measures that capture the *cumulative* distributional behavior of assessments relative to the *cumulative* distributional behavior of prices across ordered price levels. Both indexes are summary measures that are simple to calculate without regression, although there are regression-equivalent formulations that are used to statistically test for vertical equity. Because Gini-based measurements of inequality have a long history in economics, their introduction to property assessment aligns the measurement and interpretation of vertical equity with its application in other fields.

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Introduction

Vertical equity is a factor in ensuring that assessment systems are equitable and a primary reporting requirement for assessment jurisdictions (IAAO 2013). For many years, the PRD was the only measure used for vertical equity because of its simplicity of interpretation and calculation. In the literature, with the ease of running regressions with statistical software, alternatives to the PRD are available. Some commonly known regression tests are the price-related bias (PRB) by Gloudemans (2011), the two-stage least squares approach by Clapp (1990), the piece-wise regression of Sunderman et al. (1990), and linear regression tests with assessment and price as regression variables (see, e.g., IAAO 1978; Bell 1984; Paglin and Fogerty 1972; Kochin and Parks 1984; and Cheng 1974).

This paper shows how Gini-based tests for inequality, commonly used in the socio-economic literature, can be applied to property assessments. The tests for vertical equity are the Kakwani Index (KI) of Kakwani (1977), which is based on the difference of Gini measures between assessment and price, and the Modified Kakwani Index (MKI) of Fukeshige (2012), which is based on the ratio. An important distinction between both tests and the PRD and currently used measures is that Gini-based analyses do not use sales ratios (assessment/price ratios) or assessment–price regressions, which basically capture the behavior of the appraisal errors. Instead they are based on measures that capture the *cumulative* distributional behavior of assessments and prices across ordered price levels. Both the KI and MKI are easy to calculate in Excel without regression, although both have a regression-equivalent formulation for the purpose of inference using the standard regression coefficient t test.

While both test for vertical equity, the KI and MKI are formulated differently and therefore capture different properties of the assessment–price relationship. The KI measures the *difference* in how the assessment inequality curve behaves relative to the price equality curve. In contrast, the MKI is a ratio and captures the *proportional change* of the assessment inequality curve relative to the price curve.

Discussed later in the paper is why capturing proportional changes makes the MKI the preferable measure when the indexes are applied across populations (or across neighborhoods, say) in which the price distribution differs. Also discussed is how both the KI and MKI address the bias in the slope coefficient of the vertical equity regression when measurement errors are known to exist.

The Gini-based indexes, widely used to determine the regressivity or progressivity of income taxes, health benefits, and education spending to name a few applications, are introduced here for the first time in the literature of property assessment. Thus, the main contribution of this paper is the introduction of an index whose calculation and interpretation of vertical equity in property valuation are consistent with its application in the economic literature.

With a long history in the social sciences, the inequality curves, called the *Lorenz curves*, can be visualized to imply the pattern of regressivity or progressivity in property assessments. Global versus local regressivity or progressivity can be inferred from the position and shape of the assessment curve relative to the price curve and can inform on improvements to the assessment model.

This paper explains in detail the calculation of KI and MKI, the proposed vertical indexes. The next section, Visualizing Inequality, introduces the Lorenz curve to the property assessment literature. Then the section Measuring Inequality explains the steps in calculating the Gini coefficient and summarizes the inequality reflected in the price Lorenz curve. Following is the section, Measuring Relative Inequality, which introduces the concept of *relative inequality* captured by tracing how cumulative shares of assessments move across the price distribution. The relative inequality curve is called the Concentration curve. The section explains the steps in calculating the Concentration Index (CI), which summarizes the *relative* inequality in the assessment Concentration curve. Note the parallel: the *Gini coefficient* is the *inequality* measure for the *price Lorenz curve*, and the CI is the *relative inequality* measure of the *assessment Concentration curve*. The KI is the *difference* of the assessment CI to the price Gini coefficient, while the MKI is the *ratio* of the assessment CI to the price Gini coefficient. Measuring Vertical Equity discusses at length the formulation, estimation, and properties of both tests. An empirical example is presented in the next section, Empirical Application, and the last section is the Summary.

Visualizing Inequality: The Lorenz Curve

How is inequality visualized? First, define how a curve looks under complete equality, and then define the *deviation of another curve from the equality curve as a measure of inequality*.

These curves, called *Lorenz curves*, are constructed to measure inequality based on ranking a *wellness* variable from lowest to highest. The wellness variable is the base variable against which the regressivity or progressivity of a program variable is measured. In the literature, income is a typical wellness variable against which different programs are measured. For example, taxes are a program variable whose regressivity or progressivity is measured against the wellness variable income.

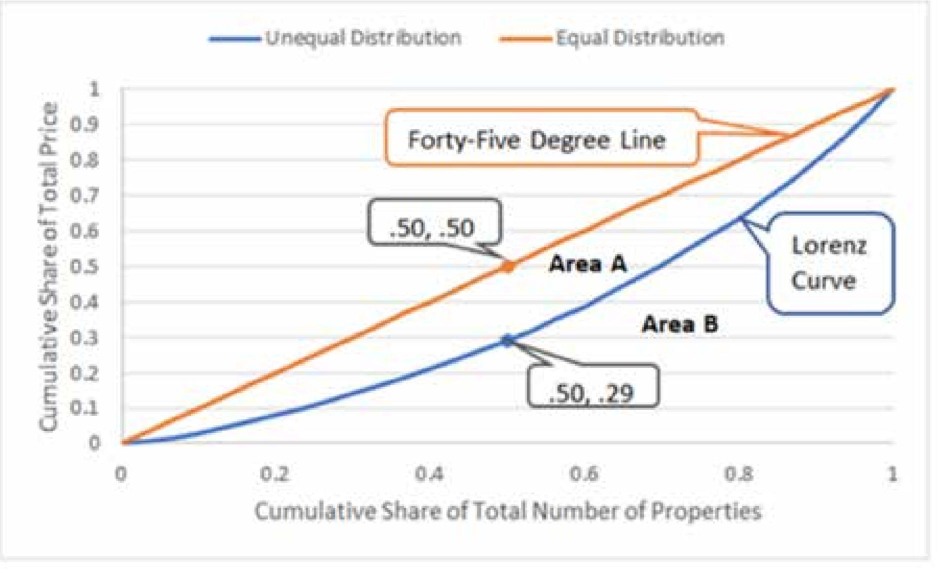
Similarly, expenditure on health is a program variable whose regressivity or progressivity can be measured against the wellness variable income. In this case, property assessments (or property taxes) is the program variable whose regressivity or progressivity is measured against the levels of real estate prices. Since real estate typically accounts for the largest share of an individual's assets, it is a proxy for wealth and thus the wellness variable in this inequality analysis.

How can Lorenz curves be constructed from real estate prices? Note that the analysis of Lorenz curves can be done by neighborhood, city, county, or any stratification chosen by a jurisdiction. Figure 1 depicts two Lorenz curves. The y axis is the cumulative share of price graphed against the x axis, which is the cumulative share of the total number of properties in the area of study. Both x and y axes range from 0 to 1 because they are calculated as percentages.

The cumulative percentages or shares are increasing from 0 to 1 because the observations are ranked from lowest to highest prices. The shares start at 0 and continually increase to 1 because all prices in the y axis and all properties in the x axis get summed. It is important to rank prices from lowest to highest prior to calculating the cumulative percentages to ensure the line is increasing from 0 to 1.

The line labeled Equal Distribution is a 45° line from the origin. The 45° line represents perfect equality, in which each property has the same price. In figure 1, for example, 50 percent of the properties in the area account for 50 percent of the total real estate values, and this equality holds across all points on the 45° line. The 45° line is the ideal in which there is no wealth distribution in terms of real estate prices. Statistically, prices are perfectly homogeneous with no skew toward low or high prices because all prices in the area are the same.

Figure 1. Lorenz curve of price distribution



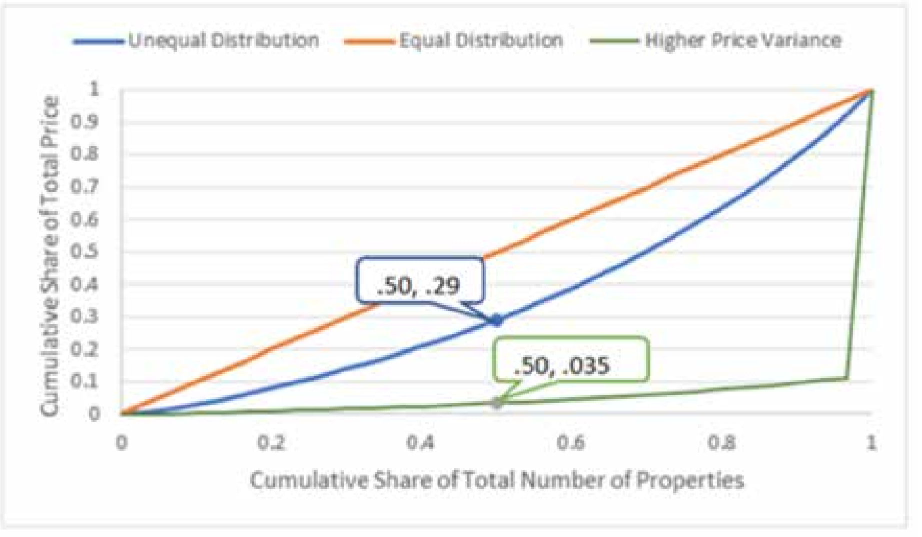
The second Lorenz curve in figure 1, labeled Unequal Distribution, results from properties in this location having different prices. The further the curve lies from the equality line or area A is larger, the higher the price inequality or price dispersion. In fact, in this case with the curve lying *below* the equality line, it is skewed with majority of the properties having lower prices and a few having the outlier higher prices. As shown, because prices are sorted from *lowest to highest*, 50 percent of the low-priced properties make up only 29 percent of the total real estate values. The other half of the properties toward the high-priced properties capture the majority of the value (71 percent). The price distribution is skewed toward high-value properties with a few capturing a large share of value. The further the curve falls below the equality line, the higher the inequality because the majority of the low-priced properties make up less of the total real estate value. A majority of the value is captured by less than half of the properties.

Figure 2 illustrates how to visualize inequality curves of price distributions with different variance and skew. Two curves that fall below the 45° line with different statistical properties are compared. Statistically, the price distribution with a higher variance and greater skew indicates a more unequal distribution. This inequality, uncaptured by the variance, is captured by the position and shape of the Lorenz curve. As a Lorenz curve, a higher inequality distribution is generally a Lorenz curve that falls further from the 45° equality line. This is denoted by the curve in figure 2 labeled Higher Price Variance, on which 50 percent of the *low*-priced properties make up only 3.5 percent of the total value or, conversely, 50 percent of the *high*-priced properties account for 96.5 percent of total real estate values.

Note that while Lorenz curves are affected by the standard measures of dispersion, such as variance and skew, they capture different properties of the distribution. Lorenz curves are constructed from *ordered* observations unlike the variance and skew, whose measurements are not dependent on observation ranks. Thus, while the variance and skew affect the Lorenz curve, the shape and position of the Lorenz curve give additional insight to price inequality by showing how prices cluster and disperse across price levels.

Table 1 shows the data required to construct the Lorenz curve in figure 1 with a sample

Figure 2. Visualizing inequality curves



of 30 sold properties. The following are the detailed steps necessary to construct and visualize inequality curves.

First, create the unequal price Lorenz curve by constructing the following columns:

1. Columns A, A', and B: *rank* price in column B from lowest to highest. Construct column A', which is the fractional rank by taking the ratio of column A divided by 30, the total number of observations.

2. Columns C and D: calculate the fractional property count (1/30) and price as a fraction of the sum total (column B, $4,047,650).

3. Columns E and F: calculate the cumulative sums of columns C and D, in which each row is the sum of the previous rows.

Second, create the equal price 45° line by constructing additional columns:

4. Column G: create a column of equal prices.

5. Column H: calculate the share of a property's price to the total price (column G, $2,250,000).

6. Column I: calculate the cumulative sums of column H by summing the previous rows.

Finally, create the Lorenz curves as shown in figure 1:

7. Graph the y axes columns (F) and (I) against the x axis (E).

These steps describe how to visualize inequality, but how is inequality measured? The following section discusses how to construct a measure of inequality using areas A and B in figure 1.

Table 1. Price data

		Unequal Price Distribution					Equal Price Distribution Forty-Five Degre Line		
Rank (A)	Fractional Rank (R') (A')	Price ($) (B)	% of Total Number of Properties (C)	% of Total Price (SP) (D)	Cumulative Share of Total Number of Properties (E)	Cumulative Share of Total Price (F)	Equal Price ($) (G)	% of Total Price (H)	Cumulative Share of Total Price (I)
1	0.0333	32,900	0.03333	0.0081	0.0333	0.0081	75,000	0.0333	0.0333
2	0.0667	36,000	0.03333	0.0089	0.0667	0.0170	75,000	0.0333	0.0667
3	0.1000	54,000	0.03333	0.0133	0.1000	0.0304	75,000	0.0333	0.1000
4	0.1333	64,500	0.03333	0.0159	0.1333	0.0463	75,000	0.0333	0.1333
5	0.1667	68,000	0.03333	0.0168	0.1667	0.0631	75,000	0.0333	0.1667
6	0.2000	70,000	0.03333	0.0173	0.2000	0.0804	75,000	0.0333	0.2000
7	0.2333	74,000	0.03333	0.0183	0.2333	0.0987	75,000	0.0333	0.2333
8	0.2667	80,000	0.03333	0.0198	0.2667	0.1184	75,000	0.0333	0.2667
9	0.3000	84,900	0.03333	0.0210	0.3000	0.1394	75,000	0.0333	0.3000
10	0.3333	89,000	0.03333	0.0220	0.3333	0.1614	75,000	0.0333	0.3333
11	0.3667	94,250	0.03333	0.0233	0.3667	0.1847	75,000	0.0333	0.3667
12	0.4000	99,000	0.03333	0.0245	0.4000	0.2091	75,000	0.0333	0.4000
13	0.4333	105,900	0.03333	0.0262	0.4333	0.2353	75,000	0.0333	0.4333
14	0.4667	109,000	0.03333	0.0269	0.4667	0.2622	75,000	0.0333	0.4667
15	0.5000	115,000	0.03333	0.0284	0.5000	0.2907	75,000	0.0333	0.5000
16	0.5333	124,500	0.03333	0.0308	0.5333	0.3214	75,000	0.0333	0.5333
17	0.5667	129,900	0.03333	0.0321	0.5667	0.3535	75,000	0.0333	0.5667
18	0.6000	135,000	0.03333	0.0334	0.6000	0.3869	75,000	0.0333	0.6000
19	0.6333	149,000	0.03333	0.0368	0.6333	0.4237	75,000	0.0333	0.6333
20	0.6667	155,800	0.03333	0.0385	0.6667	0.4622	75,000	0.0333	0.6667
21	0.7000	163,500	0.03333	0.0404	0.7000	0.5026	75,000	0.0333	0.7000
22	0.7333	175,000	0.03333	0.0432	0.7333	0.5458	75,000	0.0333	0.7333
23	0.7667	179,000	0.03333	0.0442	0.7667	0.5900	75,000	0.0333	0.7667
24	0.8000	185,600	0.03333	0.0459	0.8000	0.6359	75,000	0.0333	0.8000
25	0.8333	199,900	0.03333	0.0494	0.8333	0.6852	75,000	0.0333	0.8333
26	0.8667	215,000	0.03333	0.0531	0.8667	0.7384	75,000	0.0333	0.8667
27	0.9000	235,000	0.03333	0.0581	0.9000	0.7964	75,000	0.0333	0.9000
28	0.9333	250,000	0.03333	0.0618	0.9333	0.8582	75,000	0.0333	0.9333
29	0.9667	279,000	0.03333	0.0689	0.9667	0.9271	75,000	0.0333	0.9667
30	1.00	295,000	0.03333	0.0729	1.0000	1.0000	75,000	0.0333	1.0000
Total Price:		4,047,650					2,250,000		

Measuring Inequality: The Gini Coefficient

The Gini coefficient is a type of measure of statistical dispersion between two distributions that has become a standard measure of inequality when distributions diverge. It is a single statistical measure of inequality, ranging from 0 (complete equality when distributions are the same) to 1 (complete inequality when distributions have maximum dispersion between them). The Gini coefficient was first introduced by Corrado Gini in a 1912 book published in Italian titled *Variabilità e Mutabilità* (*Variability and Mutability*) as a measure of dispersion for *ordered* observations, unlike standard measures of dispersion like the variance in which order does not matter.

It has since been a standard measure of inequality in economics with applications that include measuring unequal income levels, unequal health benefits, unequal educational expenditures, or, more generally, social science applications concerned with the unequal effects of social programs. This paper extends the current literature by applying the Gini coefficient to the measurement of the inequalities in the administration of property valuation. Property values are the base values from which property taxes are calculated.

The Gini coefficient is the ratio of area A, the area between the Lorenz curve and the 45° line, and area A + B, the triangular area underneath the 45° line. The area ratio is equivalent to the ratio of the covariance and the mean (see O'Donnell et al. 2016 for the use of the covariance formula):

$$\text{Gini} = \frac{\textit{Area A}}{\textit{Area A} + \textit{Area B}}$$

$$= \frac{2 \times \textit{Covariance}\ (R, SP)}{\textit{Average}\ (SP)},$$

where R is the fractional rank (column A') and SP is the share of prices (column D) in table 1. (The covariance formula in the Gini equation is the population, not sample, formula taken over all the observations. For large samples, the use of the sample or population formula has a negligible effect on the Gini estimate.)

Properties of the Gini coefficient are as follows:

• Range lies in [0, 1]. The Gini coefficient is 0 with perfect equality and 1 with perfect inequality. Geometrically, a Gini at 0 is the result of the Lorenz curve lying on the equality line so that area A is equal to 0. Similarly, a Gini at 1 is the result of the Lorenz curve being furthest away from the equality line with the triangle having the maximum area of 1/2. (The area of the triangle is 1/2 (base × height). Maximum inequality is the largest triangle with base = 1 and height = 1.) Thus, at perfect inequality,

$$\text{Gini} = \frac{1/2}{1/2}$$

$$= 2 \times (1 \div 2) = 1$$

Perfect inequality is a degenerate case of figure 2 in which the highest priced property accounts for all the value (an extreme high-value outlier). At perfect inequality, the Lorenz curve lies on the x axis except at the last point where the highest priced property accounts for all the value.

• Index is unitless. The Gini coefficient, like the well-known correlation coefficient, does not depend on units of measure. This is an important property of an index because the index can be used across neighborhoods with different price levels. For example, a neighborhood with a Gini of 0.2 indicates more equal prices than another neighborhood with a Gini of 0.8, regardless of whether the comparison is made between the poorest and richest neighborhoods. The Gini coefficient does not depend on the dollar level, so the numeric interpretation remains the same when areas with different price levels are being compared.

The Gini coefficient for this sample of 30 sold properties is

$$\text{Gini} = \frac{2 \times \textit{Covariance}\ (R, SP)}{\textit{Average (SP)}}$$

$$= \frac{2 \times 0.004808}{.03333} = 0.288506.$$

The Gini coefficient is 0.28851, or 28.85 percent, which, being less than 50 percent, denotes a more equal than unequal price distribution. (Calculation of the Gini coefficient using the ratio area A ÷ (area A + area B) is given in the Appendix. It was only in the 1980s that the covariance formulation was discovered as a direct transformation of the area formula. The geometric calculation of the Gini coefficient as areas beneath the curve was the traditional method of calculating the Gini coefficient. It is an intuitive approach to understanding the reason for its inception and the behavior of the coefficient.)

Inequality is directly related to a higher variance and skew in price levels. For example, in table 1 the highest priced property is \$295,000. Suppose instead the sample has one outlier multimillion-dollar property. Consider an extreme case in which the highest price is \$29,500,000, rather than \$295,000. When the last observation is replaced with a price of \$29,500,000, the Gini calculation is

$$\text{Gini} = 2 \times \frac{\textit{Covariance}\ (R,\ SP)}{\textit{Average}\ (SP)}$$

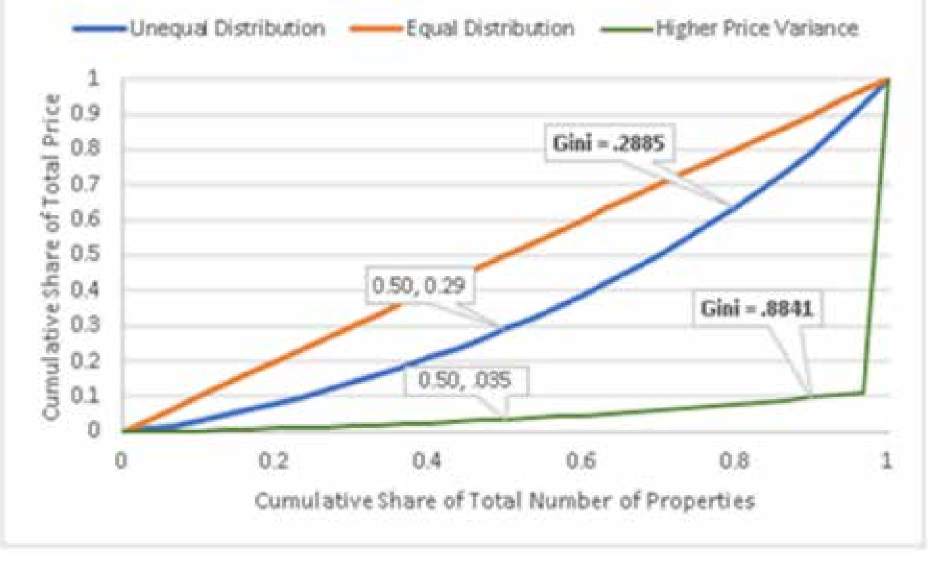
$$= \frac{2 \times 0.014735}{0.03333} = 0.884118.$$

The Gini coefficient increases to 88.41 percent from 28.85 percent, denoting a highly unequal distribution driven by a single extreme outlier.

Geometrically, the higher Gini coefficient translates to a Lorenz curve further below the 45° line. This is illustrated in figure 3 with the Lorenz curve labeled with a Gini coefficient of 0.8841. The higher variance and skew translate to higher inequality and are measured by a Gini coefficient approaching 1. Note that at midpoint, 50 percent of the low-priced properties make up only 3.5 percent of total real estate values. This distribution is more unequal to that in which 50 percent of the low-priced properties accounted for 29 percent of the total real estate values and with inequality measured by a lower Gini coefficient of 28.85 percent.

While the Gini coefficient is influenced by the standard measures of dispersion, such as the variance, additional information is captured by the ordering of prices prior to analyzing the Gini coefficient's properties. The Gini coefficient informs on the clustering and dispersion of prices across price levels, a pattern that the standard variance cannot capture. The interpretation of this statistical pattern is the inequality in prices or wealth.

Figure 3. Gini coefficients showing different degrees of inequity



Measuring Relative Inequality: The Concentration Index

How is the relative inequality of assessments to prices measured? Recall that to measure the inequality in the price distribution, there are two steps: (1) construct the Lorenz curve and (2) estimate the Gini coefficient as areas below the curve. Measuring the inequality in the assessment distribution also consists of two steps: (1) construct what is referred to as a Concentration curve and (2) estimate the CI as areas below the curve.

Note the parallel: the inequality measure of the price distribution is the *Gini coefficient,* which is calculated from the *Lorenz curve*, while the relative inequality measure of the assessment distribution across price levels is the *CI,* which is calculated from the *Concentration curve*. The Concentration curve, because it measures *relative* inequality, is the bivariate analogue of the Lorenz curve.

Table 2 adds assessment data to table 1. Note that assessment data (column B') corresponds to ordered prices. The rank (column A) and fractional rank (column A') correspond to ordered prices, not ordered assessments.

Visualizing Relative Inequality: The Concentration Curve

The Concentration curve and CI are graphed and calculated with the same procedures and formulas as the Lorenz curve and Gini coefficient. The difference is that the Concentration curve and CI are calculated for assessments, whose corresponding prices are ranked from lowest to highest levels.

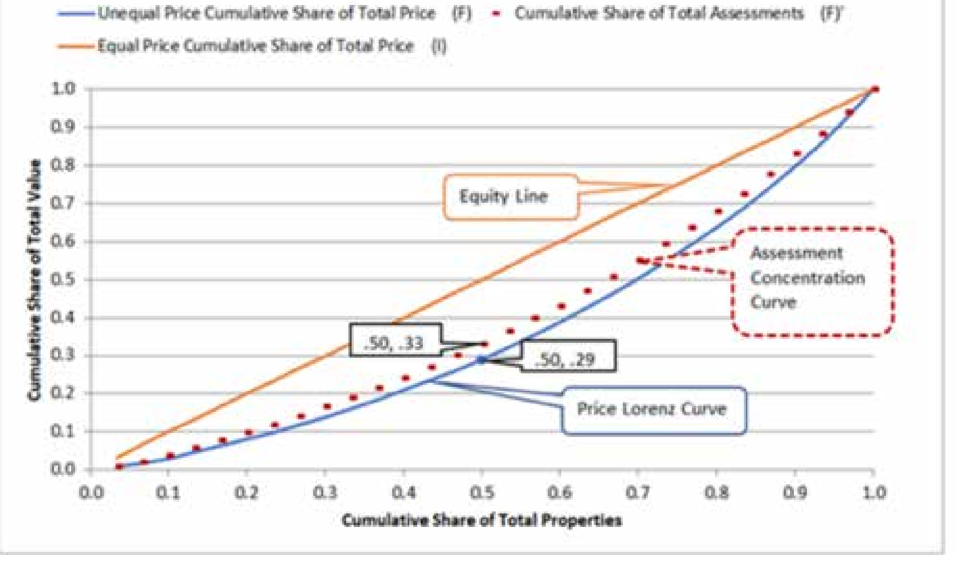
Figure 4 shows how to visualize the Lorenz and Concentration curves with data from table 2. The Concentration curve is added to the Lorenz and equality line plot by including column (F').

The Concentration curve shows how assessments "concentrate" around different price levels. At midpoint in figure 4, 50 percent of the properties account for 33 percent of assessments even though they are worth only 29 percent of total value. Thus, the position and shape of the Concentration curve relative to the Lorenz curve are more meaningful than their relation to the equality line. When the assessment curve lies above the price Lorenz curve, then low-priced properties have an increasingly higher share of assessments than they have of the share of value.

Table 2. Assessment and price data

		Assessment and Price Distribution								
Rank (A)	Fractional Rank (A')	Ranking Variable: Price (B)	Program Variable: Assessments (B')	% of Total Number of Properties (C)	% of Total Price (SP) (D)	% of Total Assessments (SA) (D')	Cumulative Share of Total Number of Properties (E)	Unequal Price Cumulative Share of Total Price (F)	Cumulative Share of Total Assessments (F')
1	0.0333	32,900	37,299	0.03333	0.0081	0.0106	0.0333	0.0081	0.0106
2	0.0667	36,000	40,166	0.03333	0.0089	0.0114	0.0667	0.0170	0.0221
3	0.1000	54,000	56,317	0.03333	0.0133	0.0160	0.1000	0.0304	0.0381
4	0.1333	64,500	66,184	0.03333	0.0159	0.0188	0.1333	0.0463	0.0570
5	0.1667	68,000	69,487	0.03333	0.0168	0.0198	0.1667	0.0631	0.0767
6	0.2000	70,000	71,515	0.03333	0.0173	0.0204	0.2000	0.0804	0.0971
7	0.2333	74,000	75,338	0.03333	0.0183	0.0215	0.2333	0.0987	0.1186
8	0.2667	80,000	81,036	0.03333	0.0198	0.0231	0.2667	0.1184	0.1416
9	0.3000	84,900	85,673	0.03333	0.0210	0.0244	0.3000	0.1394	0.1660
10	0.3333	89,000	85,021	0.03333	0.0220	0.0242	0.3333	0.1614	0.1903
11	0.3667	94,250	90,046	0.03333	0.0233	0.0256	0.3667	0.1847	0.2159
12	0.4000	99,000	94,089	0.03333	0.0245	0.0268	0.4000	0.2091	0.2427
13	0.4333	105,900	100,227	0.03333	0.0262	0.0285	0.4333	0.2353	0.2713
14	0.4667	109,000	103,157	0.03333	0.0269	0.0294	0.4667	0.2622	0.3006
15	0.5000	115,000	108,290	0.03333	0.0284	0.0308	0.5000	0.2907	0.3315
16	0.5333	124,500	117,099	0.03333	0.0308	0.0334	0.5333	0.3214	0.3648
17	0.5667	129,900	115,347	0.03333	0.0321	0.0329	0.5667	0.3535	0.3977
18	0.6000	135,000	119,678	0.03333	0.0334	0.0341	0.6000	0.3869	0.4318
19	0.6333	149,000	131,631	0.03333	0.0368	0.0375	0.6333	0.4237	0.4693
20	0.6667	155,800	137,321	0.03333	0.0385	0.0391	0.6667	0.4622	0.5084
21	0.7000	163,500	143,974	0.03333	0.0404	0.0410	0.7000	0.5026	0.5494
22	0.7333	175,000	153,572	0.03333	0.0432	0.0437	0.7333	0.5458	0.5931
23	0.7667	179,000	148,457	0.03333	0.0442	0.0423	0.7667	0.5900	0.6354
24	0.8000	185,600	153,488	0.03333	0.0459	0.0437	0.8000	0.6359	0.6791
25	0.8333	199,900	165,040	0.03333	0.0494	0.0470	0.8333	0.6852	0.7261
26	0.8667	215,000	176,940	0.03333	0.0531	0.0504	0.8667	0.7384	0.7765
27	0.9000	235,000	192,959	0.03333	0.0581	0.0550	0.9000	0.7964	0.8315
28	0.9333	250,000	180,046	0.03333	0.0618	0.0513	0.9333	0.8582	0.8827
29	0.9667	279,000	200,240	0.03333	0.0689	0.0570	0.9667	0.9271	0.9398
30	1.000	295,000	211,445	0.03333	0.0729	0.0602	1.0000	1.0000	1.0000
Totals:		4,047,650	3,511,083						

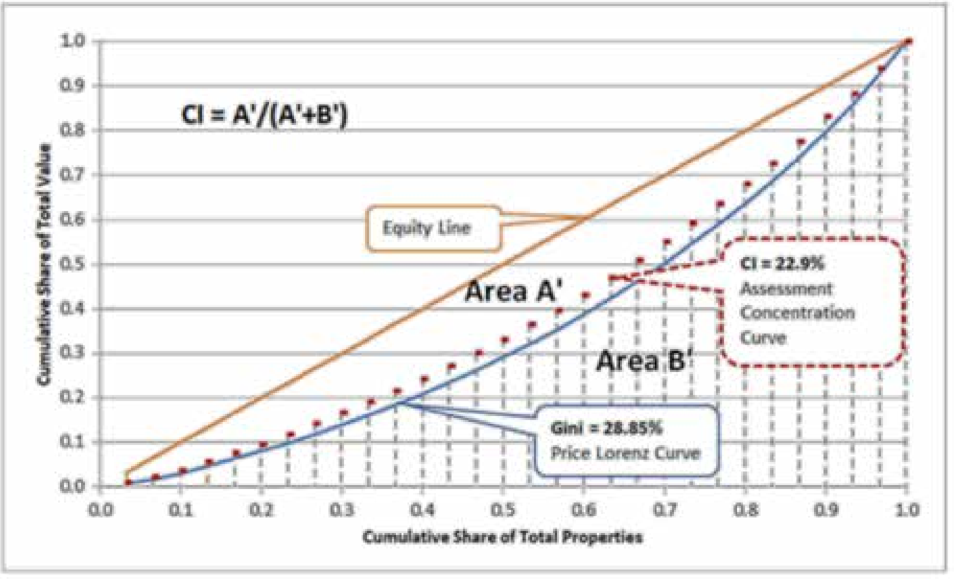
Figure 4. Concentration curve and Lorenz curve



Estimating the Concentration Index

The CI is the summary measure of *relative* inequality for the Concentration curve, comparable to how the Gini coefficient is the summary measure of inequality for the Lorenz curve. The formula is the same as the Gini coefficient except that it is done on assessments whose sequence depends on ranking prices. Note that the ranking or sequence of observations is important because the inequality measure depends on cumulative sums of ranked observations. The areas are relative to the assessment curve, as shown in figure 5.

Figure 5. The Concentration Index and the Gini coefficient



The CI is the ratio of area A', the area between the Concentration curve and the 45° line, and the sum of area A' and area B', the triangular area underneath the 45° line. It is defined as

$$\text{Cl} = \frac{\textit{Area } A'}{\textit{Area } A' + \textit{Area } B'}$$

$$= \frac{2 \times \textit{Covariance}\ (R, SA)}{\textit{Average}\ (SA)}$$

where R is the fractional rank (column A') and SA is the share of assessments (column D') in table 2.

The CI for the sample of 30 sold properties is calculated as

$$\text{Cl} = \frac{2 \times \textit{Covariance}\ (R, SA)}{\textit{Average}\ (SA)}$$

$$= \frac{2 \times 0.003816}{0.03333} = 0.228958$$

The CI is 0.228958, or 22.9 percent.

The CI makes sense only when interpreted relative to the Gini coefficient. Because both are calculated using the Gini coefficient formula, both the CI of 22.9 percent and the Gini coefficient of 28.85 percent are areas from the 45° equality line. This is shown in Figure 5 in which the formula for CI is the same as that for the Gini in terms of areas to the 45° line. Note the relation between the indexes and curves: a *lower* CI of 22.9 percent versus a Gini of 28.85 percent corresponds to the Concentration curve that lies *above* the Lorenz curve.

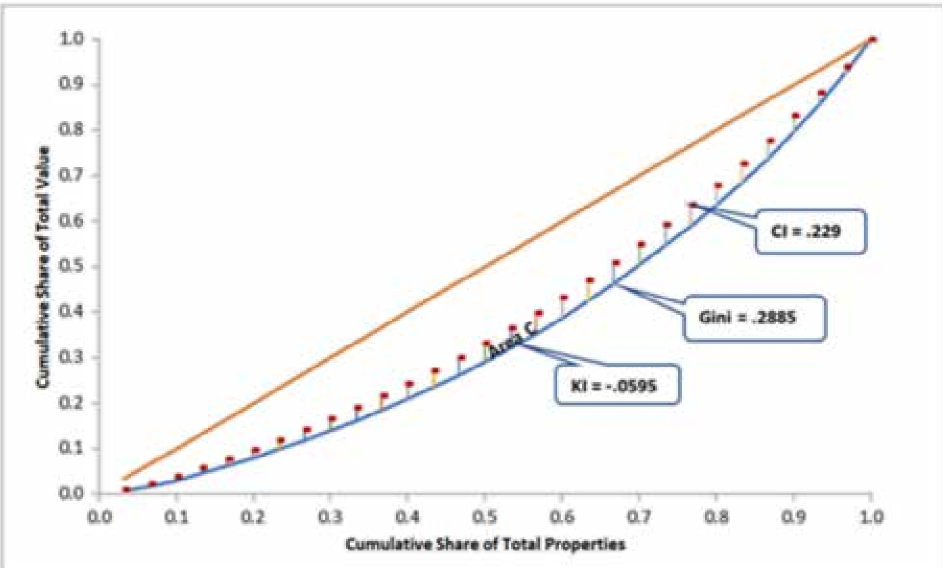
Measuring Vertical Equity: The Kakwani Index

The position and shape of the Concentration curve and the Lorenz curve determine the regressivity or progressivity of assessments. The numerical measure that summarizes the relative position of both curves is the KI (Kakwani 1977):

$$\text{KI} = \text{CI} - \text{Gini},$$

which is simply the difference between the inequality coefficients with range [−1, 1]. Geometrically, the KI measures the area between the curves; this is shown as area C in figure 6.

Figure 6. Kakwani Index



The CI is 0.229, the Gini is 0.2885, and the area between the curves is the KI, which is equal to the difference, −0.0595. A negative KI denotes a Concentration curve that lies above the Lorenz curve, as shown in figure 6.

The MKI (Fukushige, Ishikawa, and Maekawa 2012) is a simple modification of the KI, which takes the ratio of the CI to the Gini index,

$$\text{MKI} = \text{CI}/\text{Gini},$$

and is defined for Gini > 0 (a Gini equal to 0 implies prices are equal for the population of properties, which is never observed in the market). Thus, if MKI is less than 1, then the Concentration curve lies above the Lorenz curve. Conversely, if the MKI is greater than 1, then the Concentration curve falls below the Lorenz curve. With CI at 0.229 and the Gini coefficient at 0.2885, the MKI is equal to 0.79, or 79 percent, for the example in figure 6.

Table 3 summarizes the relationship between KI, MKI, and the implication for regressivity and progressivity. Assessment vertical equity or the lack of it is measured by the sign of the KI and the value of the MKI:

- *Regressivity.* Assessments are *regressive* with respect to prices if the Concentration curve lies *above* the Lorenz curve, the CI is less than the Gini coefficient (CI < Gini), the KI is negative (KI < 0), and the MKI is less than 1(MKI < 1).

- *Progressivity.* Assessments are *progressive* with respect to prices if the Concentration curve lies *below* the Lorenz curve, the CI is greater than the Gini coefficient (CI > Gini), the KI is positive (KI > 0), and the MKI is greater than 1 (MKI > 1).

- *Vertical Equity.* Assessments satisfy *vertical equity* with respect to prices if the Concentration curve lies on the Lorenz curve, the CI is equal to the Gini coefficient (CI = Gini), the KI is 0 (KI = 0), and the MKI is 1 (MKI = 1).

Table 3. Vertical equity Gini-based indexes

Assessment—Price Relationship	Kakwani Index (KI)	Modified Kakwani Index (MKI)
Regressivity	<0	<1
Progressivity	>0	>1
Vertical Equity	0	1

In figures 5 and 6, the Concentration curve lies above the Lorenz curve, so the KI is negative (−0.06, or −6 percent) and MKI = 0.79. Assessments are regressive because, while half of the low-priced properties account for 29 percent of the share of total value, their assessment share is higher at 33 percent. This is indicative of regressive assessments because low-priced properties have a higher share of the tax base than what their properties are worth.

An alternative interpretation is to note that points on the assessment curve measure the concentration of assessments across the price distribution. Assessments are regressive when properties have a share of the tax base higher than their share of total value. Conversely, assessments are progressive when properties have a share of the tax base lower than their share of total value. Note that this is a measure of vertical inequality because prices are ordered so that the assessment curve is showing the accumulation of the tax base as values increase.

Some points or segments of the Concentration curve may lie on the Lorenz curve or cross the curve at different points. Vertical equity is measured by the behavior of the entire curve and not its segments. Global versus local measures of vertical equity are discussed later in the paper.

Vertical Equity: Estimation and Inference

For large samples, there is a regression approach to estimating the CI and Gini coefficient and, by extension, testing for vertical equity. The indexes are the slope coefficients in the regressions in table 4.

Table 4. Vertical equity regression

Regression Equation	Index	Test
$2var(R)\frac{SP}{\overline{SP}} = \alpha_0 + \alpha_1 R + \epsilon$	Gini = α_1	$\alpha_1 = 0,$ $(price)\ equality$
$2var(R)\frac{SA}{\overline{SA}} = \beta_0 + \beta_1 R + \epsilon$	CI = β_1	$\beta_1 = 0,$ $relative\ equality$
$2var(R)\left(\frac{SA}{\overline{SA}} - \frac{SP}{\overline{SP}}\right) = \gamma_0 + \gamma_1 R + \epsilon$	KI = γ_1	$\gamma_1 = 0,$ $vertical\ equity$
First-stage regression: $\frac{SP}{\overline{SP}} = d_0 + d_1 R + \epsilon$ Second-stage regression using predicted: $\frac{\widehat{SP}}{\overline{SP}}$ $\frac{SA}{\overline{SA}} = \delta_0 + \delta_1 \frac{\widehat{SP}}{\overline{SP}} + \epsilon$	$MKI = \delta_1$	$\delta_1 = 1,$ $vertical\ equity$

where $\overline{SP}$ and $\overline{SA}$ are averages of SP and SA, respectively, and ϵ is the generic notation for an error term.

The MKI is a ratio measuring relative change and is estimated with a two-stage least squares approach, unlike the KI, which is based on a difference and is estimated with a simple linear regression. The two indexes have in common the use of ranks as an independent variable. These are important properties, and the following section discusses how these properties address the downward bias created by measurement errors in assessment and price.

Tables 5–8 contain the regression output run in Excel for the sample of 30 properties. The following discussion illustrates how vertical equity tests are performed using the output.

1. *Testing for Inequality in the Price Distribution Using the Gini Coefficient*

Since the *p* value for the slope coefficient is less than the significance level of 5 percent, there is no evidence of equality in the price distribution. In other words, the Lorenz curve is significantly below the 45° line as measured by area A ÷ (area A + area B), which is equal to 0.28851. See table 5.

Table 5. Equity test using the Gini regression

Regression Equation	$2var(R)\frac{SP}{\overline{SP}} = \alpha_0 + \alpha_1 R + \epsilon$
Test for equality	$\alpha_1 = 0, Gini\ coefficent$
Test statistic	Slope = 0.28851
Test result	*t* ratio = 21.79, *p* value <0.05
Test conclusion	Inequality in price distribution

Regression Statistics	
Multiple R	0.9718
R Square	0.9443
Adjusted R Square	0.9423
Standard Error	0.0209
Observations	30

ANOVA

	df	SS	MS	F	Significance F
Regression	1	0.20786	0.20786	474.99511	0.00000
Residual	28	0.01225	0.00044		
Total	29	0.22011			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	0.01742	0.00783	2.22377	0.03441	0.00137	0.03347
X Variable R	0.28851	0.01324	21.79438	0.00000	0.26139	0.31562

2. Testing for Relative Inequality of the Assessment Distribution Using the CI

Since the p value for the slope coefficient is less than the significance level of 5 percent, there is no evidence of relative equality in the assessment distribution when ordered by price levels. In other words, the Concentration curve for assessments, constructed from the order of prices, is significantly below the 45° line as measured by area A’ ÷ (area A’ + area B’), which is equal to 0.22896. See table 6.

Table 6. Relative equality test using CI regression

Regression Equation	$2var(R)\frac{SA}{\overline{SA}} = \beta_0 + \beta_1 R + \epsilon$
Test for relative equality	$\beta_1 = 0, Concentration\ Index$
Test statistic	Slope = 0.22896
Test result	*t* ratio = 34.74, *p* value < 0.05
Test conclusion	Relative inequality in assessments

Regression Statistics	
Multiple R	0.9886
R Square	0.9773
Adjusted R Square	0.9765
Standard Error	0.0104
Observations	30

ANOVA

	df	SS	MS	F	Significance F
Regression	1	0.13091	0.13091	1206.65157	0.00000
Residual	28	0.00304	0.00011		
Total	29	0.13395			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	0.04819	0.00390	12.35411	0.00000	0.04020	0.05618
X Variable R	0.22896	0.00659	34.73689	0.00000	0.21546	0.24246

3. *Testing for Vertical Equity of Assessments to Price Using the KI*

Since the p value for the slope coefficient is less than the significance level of 5 percent, there is no evidence of vertical equity in assessments. In other words, assessments are regressive since the Concentration curve is significantly above the Lorenz curve as measured by area C, which is equal to −0.05955. See table 7.

Table 7. Vertical equity test using the KI regression

Regression Equation	$2var(R)\left(\frac{SA}{\overline{SA}} - \frac{SP}{\overline{SP}}\right) = \gamma_0 + \gamma_1 R + \epsilon$
Test for vertical equity	$\gamma_1 = 0, Kakwani\ Index$
Test statistic	Slope = – 0.05955
Test result	*t* ratio = – 6.87, *p* value < 0.05
Test conclusion	Regressive assessments

Regression Statistics	
Multiple R	0.79238
R Square	0.62786
Adjusted R Square	0.61457
Standard Error	0.01369
Observations	30

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.00886	0.00886	47.24133	0.00000
Residual	28	0.00525	0.00019		
Total	29	0.01410			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.03077	0.00513	6.00099	0.00000	0.02026	0.04127
X Variable R	-0.05955	0.00866	-6.87323	0.00000	-0.07729	-0.04180

4. *Testing for Vertical Equity of Assessments to Price Using the MKI*

Since the slope coefficient in the second stage-regression is 0.79360, the test is whether the value is significantly different from 1. The *t* ratio in the regression output assumes that the test is whether the slope is different from 0; however, the test is whether the difference is 1 instead. Recalculating the test statistic as

$$t\ ratio = \frac{slope - 1}{standard\ error} = \frac{0.7936 - 1}{0.02285} = -9.03282$$

shows significance at the 5 percent confidence level. The 95 percent confidence interval can also be used for the slope coefficient. It has the range [0.74680, 84040], which is far below 1, so the slope coefficient is significantly below 1 and assessments are regressive. See table 8.

Table 8. Vertical equity test using the MKI regressions

First-Stage Regression Equation	$\frac{SP}{\overline{SP}} = d_0 + d_1 R + \epsilon$
Second-Stage Regression Equation Using Predicted $\frac{\widehat{SP}}{\overline{SP}}$	$\frac{SA}{\overline{SA}} = \delta_0 + \delta_1 \frac{\widehat{SP}}{\overline{SP}} + \epsilon$
Test for vertical equity	$\delta_1 = 1, MKI$
Test statistic	Slope = 0.79360
Test result	*t* ratio = – 9.03, *p* value < 0.05
Test conclusion	Regressive assessments

MKI First Stage Regression

Regression Statistics	
Multiple R	0.97177
R Square	0.94433
Adjusted R Square	0.94235
Standard Error	0.12565
Observations	30

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	7.49955	7.49955	474.99511	0.00000
Residual	28	0.44208	0.01579		
Total	29	7.94163			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.10464	0.04705	2.22377	0.03441	0.00825	0.20102
X Variable R	1.73296	0.07951	21.79438	0.00000	1.57008	1.89584

MKI Second Stage Regression

Regression Statistics	
Multiple R	0.98860
R Square	0.97732
Adjusted R Square	0.97651
Standard Error	0.06256
Observations	30

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	4.72321	4.72321	1206.65157	0.00000
Residual	28	0.10960	0.00391		
Total	29	4.83281			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.20640	0.02554	8.08072	0.00000	0.15408	0.25872
X Variable Predicted	0.79360	0.02285	34.73689	0.00000	0.74680	0.84040

KI versus MKI

Although the calculation of KI as a difference and MKI as a ratio is straightforward, there are implications on the interpretation and application of the tests. In particular, is one test preferred over the other? How do the tests differ in interpretation, and what do they measure? Is it possible that the KI rejects vertical equity when the MKI does not, and if so, under what conditions? Can jurisdictions set practical bounds for the tests as guidance, separate from the statistical tests provided in table 4? Understanding the properties of the tests helps in answering these questions.

The main similarities and/or differences between the MKI and KI tests are described in the following paragraphs.

First, MKI being formulated as a ratio is an elasticity index; in particular, it measures the elasticity of the shares of assessment to the shares of price (both evaluated at their respective means). Elasticity is the measurement of the percentage change in one variable in response to another. To explain this concept, an elastic variable (with elasticity greater than one) responds more than proportionally to changes in another variable. In contrast, an inelastic variable (with elasticity less than 1) changes less than proportionally in response to another variable. Formulaically, the MKI can be written as an elasticity index by rewriting the two-stage least squares MKI estimate in its instrumental variable form (with *R* as instrument):

$$\widehat{\delta_1} = \frac{\sum_i \; R_i(SA - \overline{SA})/\overline{S.}}{\sum_i \; R_i(SP - \overline{SP})/\overline{S}}$$

Note the shares correlation with the instrument *R*. Thus, as *R* increases, that is, as the price level increases, the MKI measures the percentage change (from its means) of the shares of assessments as the shares of prices change.

The KI is not an elasticity measure but a difference change measure. KI measures the difference of the Concentration curve to the Lorenz curve (or the difference of CI to the Gini coefficient). This distinction is important when the population is *heterogeneous with respect to price*, for example, jurisdictions with neighborhoods that have different patterns of extremely high and extremely low prices. If the KI is calculated by neighborhood, these neighborhoods can have the same KI even though the Lorenz curves are very different; that is, the price distributions in each neighborhood are very different. This is because the KI measures the spread between the Concentration curve and the Lorenz curve regardless of where in the spectrum the Lorenz curve lies (i.e., regardless of the heterogeneity of prices).

In contrast, the MKI measures proportional changes. Suppose the MKI is calculated for different neighborhoods with heterogeneous prices and, say, the MKI is 95 percent for all. This is interpreted as follows: regardless of the price distribution by neighborhood, when shares of (ordered) prices increase, then shares of assessment respond less than proportionally (at 95 percent) for all neighborhoods. In other words, assessments are regressive in all neighborhoods regardless of price distribution (note, statistical tests can indicate otherwise because the different errors in each neighborhood give different confidence intervals).

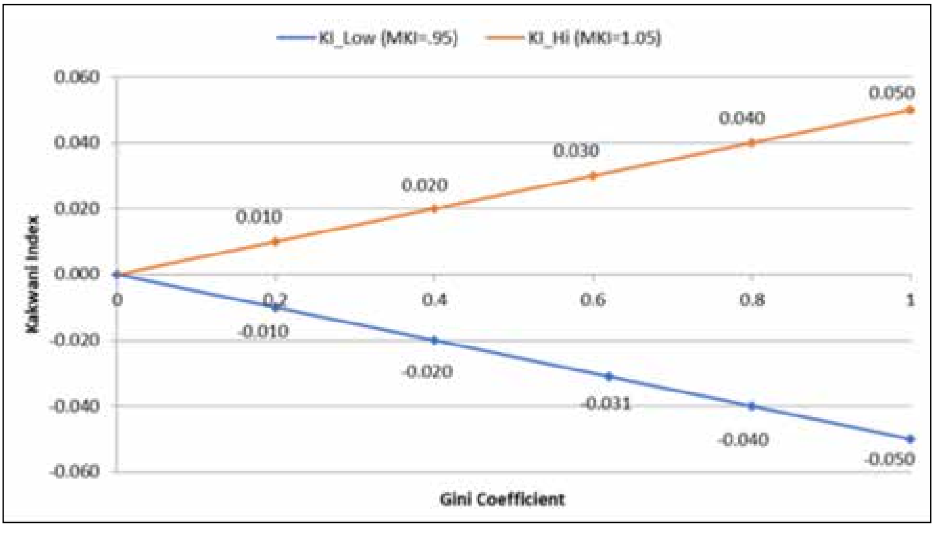
As practical guidance for jurisdictions, it is sometimes desirable to give guidance of when vertical equity is considered satisfied, *separate from vertical equity results that depend on statistical testing* (similar to the PRD rule of [0.98, 1.03] where bootstrap testing can result in different conclusions). In general, an MKI in the range [0.95, 1.05] is considered vertically equitable; that is, shares of assessments respond to within 5 percent as shares of prices change. If a jurisdiction wishes to provide further practical guidance on the admissible spread of KI, given the 5 percent rule of the MKI, the following relationship must be used:

$$KI = (MKI - 1) \times Gini.$$

In other words, a set interval on the KI requires a set rule on the Gini; that is, neighborhoods with more heterogeneity in price (a greater Gini coefficient) should be given a wider allowable KI spread.

Figure 7 illustrates the KI spreads for different Gini coefficients given MKI at its admissible lower bound of 0.95, and separately at its admissible higher bound of 1.05. To read the admissible KI spread in figure 7, first identify the Gini coefficient; for example, a neighborhood with a Gini of 0.2 has an admissible KI spread of [−0.01, 0.01]; a neighborhood with a Gini of 0.4 has an admissible KI spread of [−0.02, 0.02]; and a neighborhood with higher price variance and a Gini of 0.8 has a wider admissible KI spread of [−0.04, 0.04].

Figure 7. MKI, KI, and Gini relationship for MKI = 0.95 and MKI = 1.05



Note that *MKI and KI are different tests with different interpretation and, more importantly, are estimated by different regression approaches*. Thus, errors used to construct the confidence intervals are different. This implies vertical equity can be rejected with MKI and not KI, and vice versa. When and under what circumstances one is preferable to the other is the subject of future work.

Second, both the MKI and KI use ranks as an independent variable, and in addition, MKI uses ranks as an instrumental variable. Thus, both MKI and KI address the measurement error problem associated with assessment and price.

The literature on vertical equity discusses the downward bias of vertical equity tests because of regressions in which the independent variable, either assessment or price, is measured with error from its true market value (for discussions on the bias, see Kochin and Parks [1984], Clapp [1990], Sunderman et al. [1990], and Gloudemans [2011]). When measurement errors (also referred to as errors-in-variables) on the independent variables are known to exist, a standard econometric solution is an instrumental variable, or two-stage least squares approach. Clapp (1990) was the first to address the measurement error problem using a two-stage least squares or instrumental variable approach *and* rank order information. The Clapp (1990) two-stage regression is

First-stage regression

$$\ln A = a_0 + a_1 Z + \varepsilon$$

Second-stage regression using predicted $\widehat{\ln A}$

$$\ln P = p_0 + p_1 \widehat{\ln A} + \varepsilon$$

where

Z = –1 if rank A is in the bottom one-third and rank p is in the bottom one-third

Z = +1 if rank A and rank p are both in the top one-third

Z = 0 otherwise

The first regression is a regression of the log of assessments on the instrument Z from which the predicted log of assessment is calculated. This is followed by the second regression of the log of price on the predicted log of assessment. The test for vertical equity is that the slope coefficient in the second regression is 1; that is, $p_1 = 1$. Because log of assessment is the independent variable, a slope coefficient greater than 1 indicates regressivity and a slope less than 1 indicates progressivity.

The two-stage least squares approach was introduced by Clapp (1990) to address the measurement errors in price and assessment that are used to proxy for market value. In a regression in which the independent variable is known to be measured with error, the regression coefficient is biased downward. Thus, if assessments are assumed to be measured with error and the second-stage regression is run without the first-stage regression, then the slope coefficient is biased downward and the test for vertical equity is biased toward finding progressivity.

The use of the first-stage regression on the rank-dependent variable Z treats it as an instrumental variable. The instrumental variable, *Z*, is correlated with assessments and prices (since it is constructed from its rank) but (assumed) uncorrelated with the error (measurement error is unobserved so the correlation cannot be measured). Thus, the instrument Z minimizes or eliminates the downward bias when the second-stage regression is used with the Z information contained in the predicted assessments as the independent variable. Note that while the literature argues whether assessment or price should be the independent variable that is measured with error, this issue is minimized in the Clapp framework since the Z instrument is constructed from both (Gloudemans [2011] is another approach that minimizes the bias by constructing a value proxy as a weighted sum of both assessment and price).

For both the MKI and Clapp approaches, the use of an instrumental variable based on ranks of the independent variable ensures it is correlated with the independent variable and minimizes or eliminates correlation with the errors. Whether this occurs in data, specifically which approach minimizes the bias more, cannot be measured because the measurement error is unobservable (so its correlation to the errors cannot be calculated). A comparison is possible, however, in a well-constructed simulation framework. Certainly, at least theoretically, the two approaches should dominate other approaches that do not account for the bias in the regression formulation, but the MKI and Clapp approaches should be compared with other tests in another paper.

Note that the KI regression, at least theoretically, with the rank as the independent variable should also minimize the bias in the test for vertical equity due to measurement error. Although KI regression is not a two-stage least squares approach, the dependent variable is based on the difference, thereby cancelling out measurement errors common to both assessment and price. The question of which test, the MKI, KI, or Clapp, reduces the downward bias more is the subject of future research.

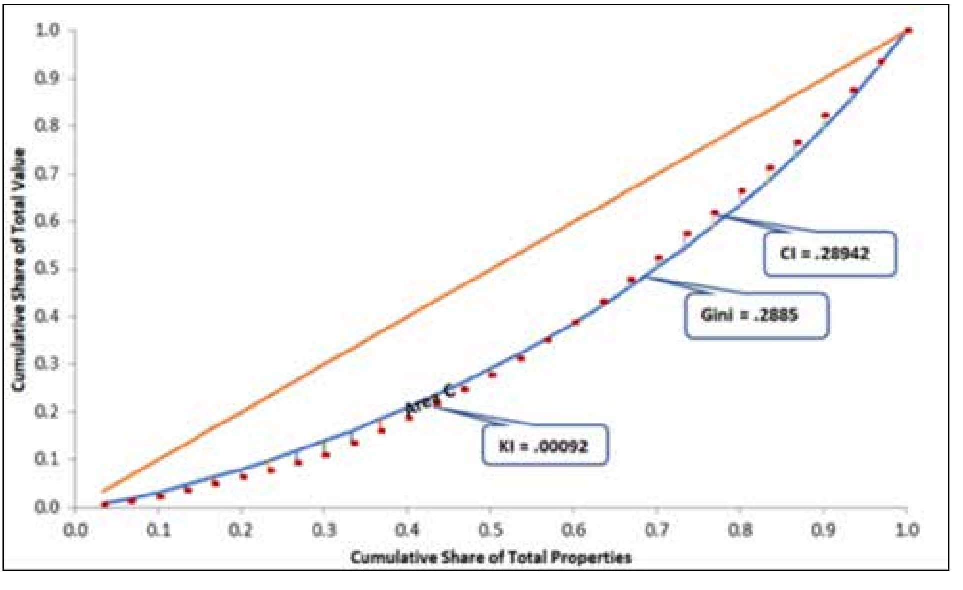
Third, both the KI and MKI are formulated using the cumulative distributions of assessments and prices. Thus they behave differently than tests constructed from the sales ratio or from coefficients of an assessment–price regression. When assessments are derived from regressions as the predicted estimates of sale prices, the error can be driven by uncaptured heterogeneity, omitted variables, errors-in-variables, incorrect functional form, or more generally a misspecified regression equation. Whether both indexes are more sensitive to these misspecifications compared to other tests in the literature requires a simulation study. As the focus of this paper is to introduce the indexes for the first time, a more in-depth study of its behavior under misspecification is the subject for another paper.

Global and Local Vertical Equity

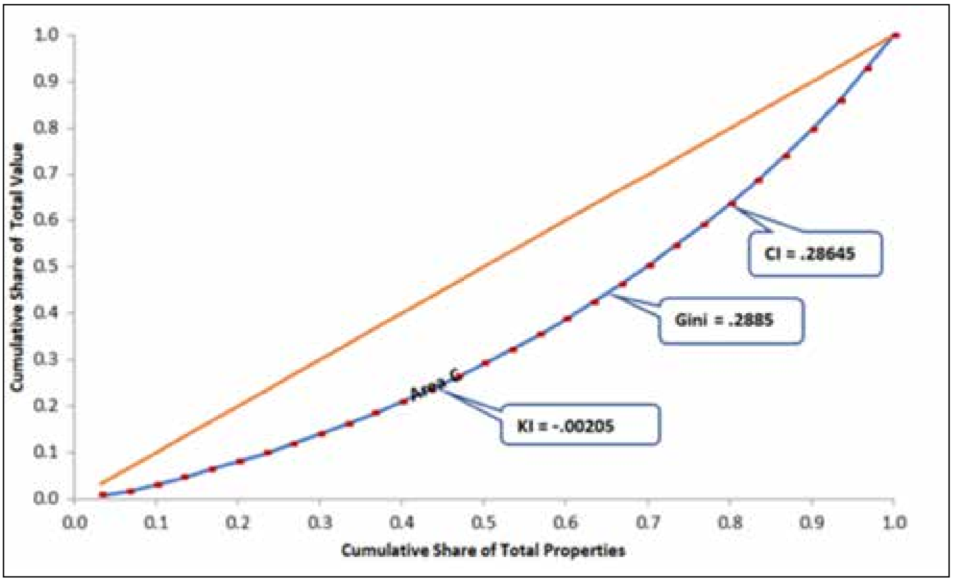
When the Concentration curve and Lorenz curve intersect, there are local and global effects between the program and ranking variables. For example, in figure 8(a), the Concentration curve lies below the Lorenz curve for lower price levels; however, the behavior changes at higher price levels. In this example assessments are locally progressive for lower price levels and become regressive for higher prices in the sense that they are assessed higher than their value. The assessment system is globally equitable with a positive KI = 0.0092 and MKI = 1.003, but regressivity and progressivity exist locally at different price levels.

Global and local vertical equity is the goal of most assessment systems in which properties are assessed at their value. In figure 8(b), the KI index is 0; MKI is 1; and all properties are assessed at their value since the Concentration curve lies on the Lorenz curve.

Figure 8. (a) Global vertical equity with local vertical inequality and (b) global and local vertical equity



(a)



(b)

The KI and MKI are formulated as tests for *global* vertical equity. To test for *local* equity, KI and MKI regressions can be performed on segments of prices by quantiles. Alternatively, a nonlinear formulation capturing curve crossings to test for *local* regressivity or progressivity can be formulated. Prior to modeling curve crossings, graphing the curves as in figures 8(a) and 8(b) gives a necessary overview of the equality in an assessment system globally and locally across different price levels.

Empirical Application

Vertical equity analyses were conducted on a dataset consisting of 4,310 sold single-family residential properties in a large jurisdiction. Prior to application of the vertical equity tests, the model was determined to satisfy statistical diagnostics for an acceptable predictive model. As shown below, the median sales ratio, defined as the predicted assessment to sale price, is 0.9971, the coefficient of dispersion (COD) is 11.49 percent, indicating acceptable error variance, and the White test for heteroskedasticity is 0.2102, indicating homoskedastic residuals.

Median Sales Ratio	0.9971
COD	0.1149
White's Test for Heteroskedasticity	0.2102

To determine vertical equity, the PRD is calculated, and regressions are performed for several tests; results are given in table 9. The PRD of 1.02 is indicative of vertical equity, unlike the regression-based tests, which uniformly indicate regressive assessments. It is possible to statistically test for the PRD using bootstrap confidence intervals rather than the [0.98, 1.03] bounds. However, the focus here is on the comparison of the regression-based tests.

Table 9. Vertical equity diagnostics

Diagnostic	Regression	Test for Equity	Coefficient	t-stat	p-value	Conclusion
PRD	N/A	[.98, 1.03]	1.0200318			Vertical Equity
ASR	$AS\ Ratio = \alpha_0 + \alpha_1 Price + \epsilon$	α_1=0	-0.0000004	-32.20	0.00000	Regressive
PRB	$RatioChange = \beta_0 + \beta_1 Ln(Value)/.693 + \epsilon$	β_1=0	-0.0906316	-16.99	0.00000	Regressive
CLAPP	$Ln(Price) = \rho_0 + \rho_1 \widehat{LN(A)} + \epsilon$	ρ_1=1	1.1853937	5.65	0.00000	Regressive
KI	$2var(R)\left(\frac{SA}{\overline{SA}} - \frac{SP}{\overline{SP}}\right) = \gamma_0 + \gamma_1 R + \epsilon$	γ_1=0	-0.0340277	-28.482	0.00000	Regressive
MKI	First stage regression: $\frac{SP}{\overline{SP}} = d_0 + d_1 R + \epsilon$ Second stage regression: $\frac{SA}{\overline{SA}} = \delta_0 + \delta_1 \frac{\widehat{SP}}{\overline{SP}} + \epsilon$	δ_1=1	0.8034160	-20.556	0.00000	Regressive

The assessment-sales ratio regression (ASR) is a regression of the sales ratio (assessment/price) on the independent variable, which is price. As shown in figure 9, the slope is negative, indicating sales ratios fall with higher prices. This is indicative of regressive assessments because ratios less than 1 are under-assessments and occur as prices increase.

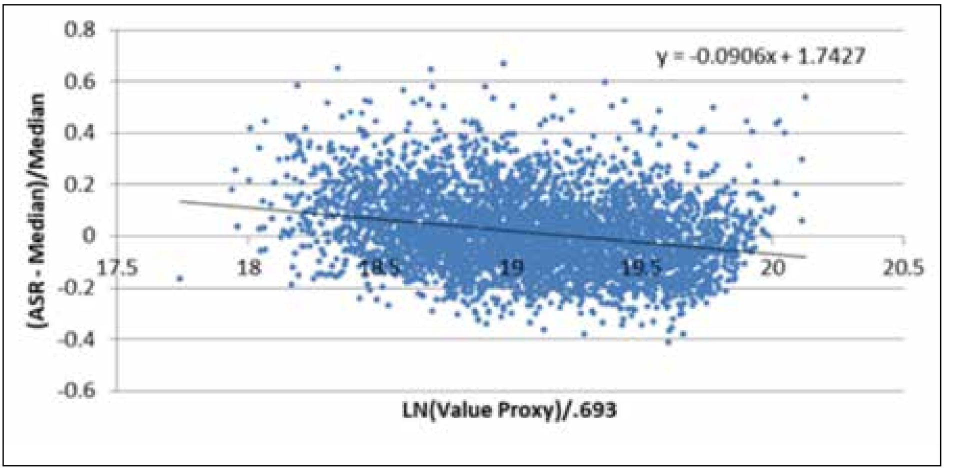
Figure 9. Vertical equity for ASR regresssion

The PRB vertical equity regression of Gloudemans (2011) is shown in figure 10. Because assessment and prices contain measurement errors to the true market value, the approach constructs a value proxy by splitting the difference between prices (P) and assessments (A) as follows:

$$Value\ Proxy = 0.50 \times P + 0.50 \times (A \div \text{median}),$$

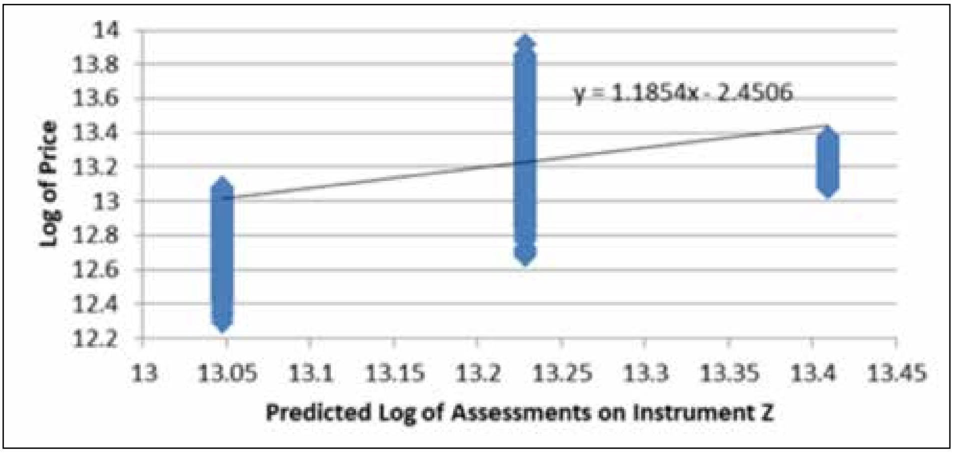
where the median is the median of ASR. The dependent variable is the percentage deviation of the sales ratio to its median, (ASR – median)/median, and the independent variable is the log base 2 of the value proxy, that is, LN(Value Proxy)/0.693. The use of the logarithmic base 2 allows for the interpretation of the regression coefficient as a percentage point of ratio increase as prices double. As with the ASR regression, the significant negative slope is indicative of regressive assessments.

Figure 10. Vertical equity for PRB regression



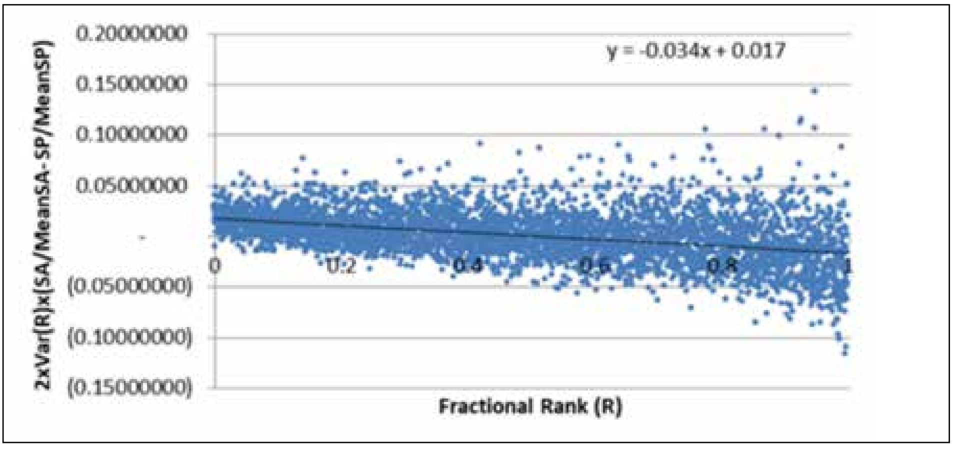
The second-stage Clapp (1990) regression is shown in figure 11. It is a regression of the natural logarithm of price to the predicted natural logarithm of assessments. The prediction is derived from a first-stage regression of the natural logarithm of assessments on the instrument *Z*. The slope coefficient of 1.1854 being greater than 1 indicates it is regressive. The test statistic recalculated around a null hypothesis of 1 indicates the distance above 1 is significant and assessments are regressive.

Figure 11. Vertical equity for Clapp regression



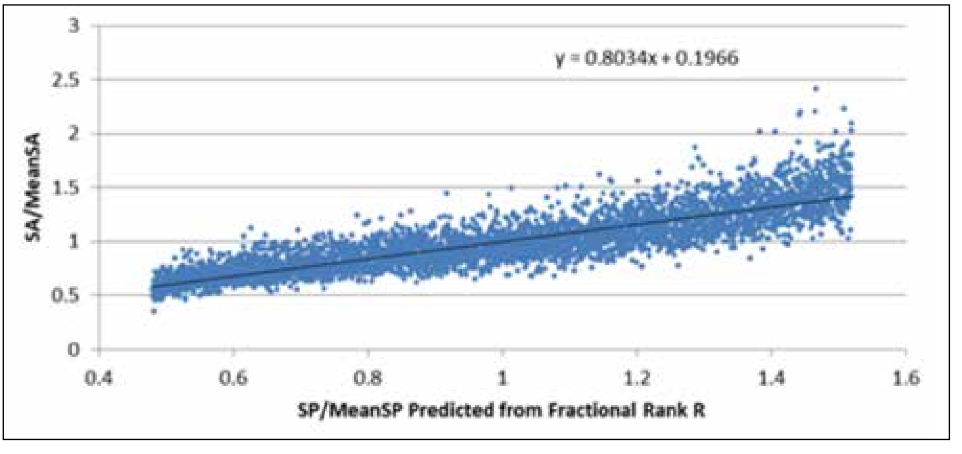
The KI regression in figure 12 is a regression with the dependent variable constructed as twice the variance of *R* multiplied by the difference in the shares of assessments divided by its mean (MeanSA) and the shares of prices divided by its mean (MeanSP). The independent variable is the fractional rank of price *R*. The slope coefficient being significantly negative is indicative of regressive assessments because, as prices increase, the difference in the shares of the assessments to the shares of prices decreases.

Figure 12. Vertical equity for KI regression



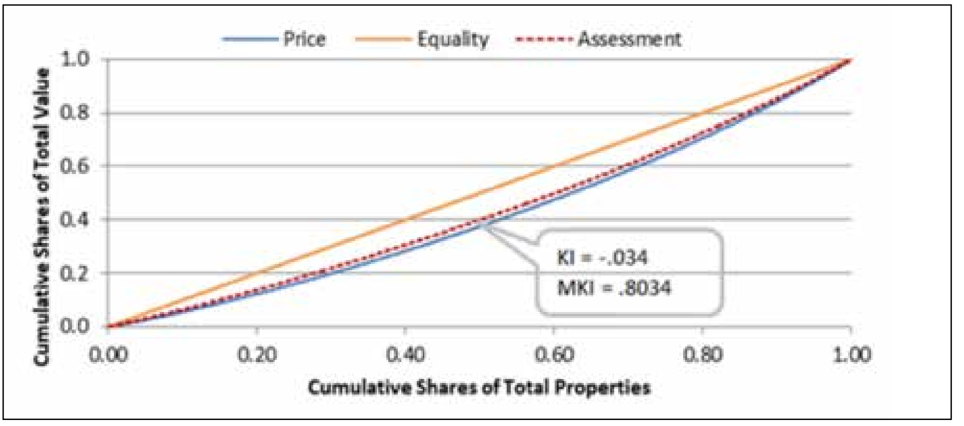
The second stage MKI regression, shown in figure 13, is a regression of the shares of assessments normalized by its mean on the predicted shares in prices normalized by its mean. The prediction is derived from a first-stage regression of shares of prices normalized by its mean on its fractional rank. The slope coefficient of 0.8034 being less than 1 indicates that assessments are regressive. The *t* statistic of −20.556 indicates the distance to 1 is statistically significant, and thus assessments are regressive.

Figure 13. Vertical equity for MKI regression



An alternative to the KI and MKI regressions in figures 12 and 13, respectively, is to visualize the inequality as Lorenz and Concentration curves as in figure 14. A negative KI of − 0.034 and an MKI of 0.8034 are illustrated as the assessment Concentration curve lying above the Lorenz curve. Graphing the inequality curves provides additional information since it shows the pattern of regressivity. In this case, regressivity is global since the Concentration curve lies everywhere above the Lorenz curve.

Figure 14. Visualizing vertical equity



Summary

Vertical equity analysis is a primary reporting requirement in property assessment (IAAO 2013). For many years the PRD was the only measure used for vertical equity because of its ease of calculation. This has changed with affordable access to regression software, providing several alternatives to the PRD. A major advantage of these regression-based tests is to easily perform statistical inference for vertical equity using standard tests given in the regression output.

The KI was originally formulated as a geometric formula of areas between curves, and then later regression equivalence was formulated. It is both a summary index measure and a regression-based measure. Furthermore, the KI has a different interpretation than the current vertical equity measures in the literature because it is not based on sales ratios. Rather, it is based on distributional differences, in this case the difference between the relative distribution of assessments and the ordered distribution of prices.

The MKI is the ratio of the CI to the Gini coefficient. Because it measures proportional changes between variables, it is an elasticity measure widely used in economics to compare heterogeneous populations. It is a summary measure like the KI, and it also has a regression equivalent that makes it possible to easily test for vertical equity.

The contributions of the KI and MKI to the currently used indexes are that (1) the calculation and interpretation of vertical equity are consistent with their application in the economic literature, and (2) the inequality curves can be visualized to infer the pattern of regressivity or progressivity. Global versus local regressivity or progressivity can be inferred visually and can inform on improvements to the assessment model.

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Appendix

Gini Calculation Using Geometry

The Gini coefficient is the area ratio A ÷ (A + B) shown in figure A-1. Note that area A + B is the area of the lower half of the 45° line. Area A + B is equal to 1/2 so the Gini formula can be rewritten in terms of only the area B:

$$\text{Gini} = A \div 0.5 = (0.5 - B) \div 0.5 = 1 - 2B. \qquad \text{(A-1)}$$

Thus by construction, because it is only a function of area B, which has a minimum of 0 and a maximum of 1/2, the Gini coefficient is bounded between 0 (complete equality when all properties have the same price) and 1 (complete inequality when price is skewed toward lower or higher prices). Boundedness is a desirable property of an index because it makes it comparable across systems.

Estimating the Gini Coefficient

The Gini coefficient is derived by estimating area B as given in equation (A-1). Consider figure A-1, which breaks area B into the sum of smaller areas $b_{\mathbf{i}}$. Area B is derived using the trapezoidal rule of summing the areas of (approximate) triangles, denoted by b_i, which is the (approximate) triangular space bounded by the dashed lines. Each triangle $b_{\mathbf{i}}$ is the product of the base and height at midpoint.

Figure A-1. Calculating the Gini coefficient as areas under a curve

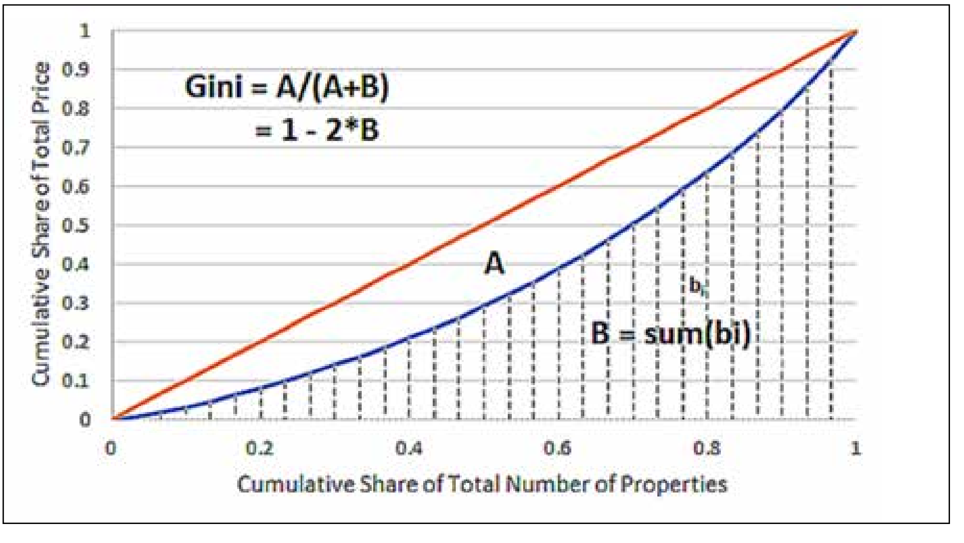

Table A-1 shows the Excel calculation: the base of the triangle is the difference in the share of the total number of properties (the x axis) denoted as column G; the height at midpoint is the average of the upper and lower points of the triangle denoted as column H; the area of the triangle is the product of the base and height at midpoint, which is column I. Finally, area B is the sum of the areas of the triangles, and the Gini coefficient is 1 less twice area B. The Gini coefficient is 0.28851, or 28.85 percent, which, being less than 50 percent, denotes a more equal than unequal price distribution.

Table A-1. Price date and formulas for calculating areas under a curve

						Gini Calculation		
Rank (A)	Price ($) (B)	% of Total Number of Properties (C)	% of Total Price (D)	Cumulative Share of Total Number of Properties (E)	Cumulative Share of Total Price (F)	Triangle Base (G) = (C)	Triangle Height at Midpoint (H) = .5*(F + max(lag(F,0)))	Area B (I) = (G) * (H)
1	32,900	0.03333	0.0081	0.0333	0.0081	0.03333	0.0041	0.00014
2	36,000	0.03333	0.0089	0.0667	0.0170	0.03333	0.0126	0.00042
3	54,000	0.03333	0.0133	0.1000	0.0304	0.03333	0.0237	0.00079
4	64,500	0.03333	0.0159	0.1333	0.0463	0.03333	0.0383	0.00128
5	68,000	0.03333	0.0168	0.1667	0.0631	0.03333	0.0547	0.00182
6	70,000	0.03333	0.0173	0.2000	0.0804	0.03333	0.0717	0.00239
7	74,000	0.03333	0.0183	0.2333	0.0987	0.03333	0.0895	0.00298
8	80,000	0.03333	0.0198	0.2667	0.1184	0.03333	0.1086	0.00362
9	84,900	0.03333	0.0210	0.3000	0.1394	0.03333	0.1289	0.00430
10	89,000	0.03333	0.0220	0.3333	0.1614	0.03333	0.1504	0.00501
11	94,250	0.03333	0.0233	0.3667	0.1847	0.03333	0.1730	0.00577
12	99,000	0.03333	0.0245	0.4000	0.2091	0.03333	0.1969	0.00656
13	105,900	0.03333	0.0262	0.4333	0.2353	0.03333	0.2222	0.00741
14	109,000	0.03333	0.0269	0.4667	0.2622	0.03333	0.2488	0.00829
15	115,000	0.03333	0.0284	0.5000	0.2907	0.03333	0.2764	0.00921
16	124,500	0.03333	0.0308	0.5333	0.3214	0.03333	0.3060	0.01020
17	129,900	0.03333	0.0321	0.5667	0.3535	0.03333	0.3375	0.01125
18	135,000	0.03333	0.0334	0.6000	0.3869	0.03333	0.3702	0.01234
19	149,000	0.03333	0.0368	0.6333	0.4237	0.03333	0.4053	0.01351
20	155,800	0.03333	0.0385	0.6667	0.4622	0.03333	0.4429	0.01476
21	163,500	0.03333	0.0404	0.7000	0.5026	0.03333	0.4824	0.01608
22	175,000	0.03333	0.0432	0.7333	0.5458	0.03333	0.5242	0.01747
23	179,000	0.03333	0.0442	0.7667	0.5900	0.03333	0.5679	0.01893
24	185,600	0.03333	0.0459	0.8000	0.6359	0.03333	0.6129	0.02043
25	199,900	0.03333	0.0494	0.8333	0.6852	0.03333	0.6606	0.02202
26	215,000	0.03333	0.0531	0.8667	0.7384	0.03333	0.7118	0.02373
27	235,000	0.03333	0.0581	0.9000	0.7964	0.03333	0.7674	0.02558
28	250,000	0.03333	0.0618	0.9333	0.8582	0.03333	0.8273	0.02758
29	279,000	0.03333	0.0689	0.9667	0.9271	0.03333	0.8927	0.02976
30	295,000	0.03333	0.0729	1.0000	1.0000	0.03333	0.9636	0.03212
Total Price:	4,047,650						Area B Total:	0.35575
							Gini :	0.28851

Inequality is directly related to a higher variance and skew in price levels. For example, in table A-1, the last observation is highlighted as the highest price in the sample at $295,000. Suppose instead the sample has one outlier multimillion-dollar property. Take an extreme case in which the highest price is $29,500,000, rather than $295,000. Compare how the Gini coefficients change because of the outlier multimillion-dollar property. The Gini coefficient, as calculated in table A-2, increases to 88.41 percent from 28.85 percent, denoting a highly unequal distribution due to one extreme observation. The higher Gini coefficient translates to a Lorenz curve further below the 45° line.

Table A-2. Effect of an outlier on the Gini coefficient

Rank (A)	Price ($) (B)	% of Total Number of Properties (C)	% of Total Price (D)	Cumulative Share of Total Number of Properties (E)	Cumulative Share of Total Price (F)	Gini Calculation: Triangle Base (G) = (C)	Gini Calculation: Triangle Height at Midpoint (H) = .5*(F + max(lag(F,0)))	Gini Calculation: Area B (I) = (G) * (H)
1	32,900	0.03333	0.0010	0.0333	0.0010	0.03333	0.0005	0.00002
2	36,000	0.03333	0.0011	0.0667	0.0021	0.03333	0.0015	0.00005
3	54,000	0.03333	0.0016	0.1000	0.0037	0.03333	0.0029	0.00010
4	64,500	0.03333	0.0019	0.1333	0.0056	0.03333	0.0047	0.00016
5	68,000	0.03333	0.0020	0.1667	0.0077	0.03333	0.0067	0.00022
6	70,000	0.03333	0.0021	0.2000	0.0098	0.03333	0.0087	0.00029
7	74,000	0.03333	0.0022	0.2333	0.0120	0.03333	0.0109	0.00036
8	80,000	0.03333	0.0024	0.2667	0.0144	0.03333	0.0132	0.00044
9	84,900	0.03333	0.0026	0.3000	0.0170	0.03333	0.0157	0.00052
10	89,000	0.03333	0.0027	0.3333	0.0196	0.03333	0.0183	0.00061
11	94,250	0.03333	0.0028	0.3667	0.0225	0.03333	0.0211	0.00070
12	99,000	0.03333	0.0030	0.4000	0.0255	0.03333	0.0240	0.00080
13	105,900	0.03333	0.0032	0.4333	0.0286	0.03333	0.0271	0.00090
14	109,000	0.03333	0.0033	0.4667	0.0319	0.03333	0.0303	0.00101
15	115,000	0.03333	0.0035	0.5000	0.0354	0.03333	0.0336	0.00112
16	124,500	0.03333	0.0037	0.5333	0.0391	0.03333	0.0373	0.00124
17	129,900	0.03333	0.0039	0.5667	0.0430	0.03333	0.0411	0.00137
18	135,000	0.03333	0.0041	0.6000	0.0471	0.03333	0.0451	0.00150
19	149,000	0.03333	0.0045	0.6333	0.0516	0.03333	0.0493	0.00164
20	155,800	0.03333	0.0047	0.6667	0.0563	0.03333	0.0539	0.00180
21	163,500	0.03333	0.0049	0.7000	0.0612	0.03333	0.0587	0.00196
22	175,000	0.03333	0.0053	0.7333	0.0664	0.03333	0.0638	0.00213
23	179,000	0.03333	0.0054	0.7667	0.0718	0.03333	0.0691	0.00230
24	185,600	0.03333	0.0056	0.8000	0.0774	0.03333	0.0746	0.00249
25	199,900	0.03333	0.0060	0.8333	0.0834	0.03333	0.0804	0.00268
26	215,000	0.03333	0.0065	0.8667	0.0899	0.03333	0.0866	0.00289
27	235,000	0.03333	0.0071	0.9000	0.0969	0.03333	0.0934	0.00311
28	250,000	0.03333	0.0075	0.9333	0.1045	0.03333	0.1007	0.00336
29	279,000	0.03333	0.0084	0.9667	0.1129	0.03333	0.1087	0.00362
30	29,500,000	0.03333	0.8871	1.0000	1.0000	0.03333	0.5564	0.01855
Total Price:	33,252,650						Area B Total:	0.05794
							Gini:	0.88412

Florida Court of Appeal Rules against Rushmore Approach in Disney Resort Decision

BY DANIEL H. LESSER

Abstract

A June 2020 decision issued by the Florida Fifth District Court of Appeal could have far-reaching implications for the market valuation of hotel properties throughout the United States. The appellate court upheld a lower court's decision rejecting the valuation methodology, the Rushmore Approach, utilized by the Orange County Property Appraiser in its assessment of the Disney Yacht & Beach Club Resort. It was 1978 when Rushmore posited the first accepted methodology for separating income attributable to business (intangible asset) from income attributable to personal property from the entire income stream of a lodging facility.

Introduction

On June 19, 2020, the Florida Fifth District Court of Appeal issued a decision that could have far-reaching implications for the market valuation of hotel properties throughout the United States. The case, *Rick Singh, As Property Appraiser*, *vs. Walt Disney Parks and Resorts US, Inc*., *et al.,* involved a tax appeal of the 2015 assessment of the Disney Yacht & Beach Club Resort in Orlando, Florida (*Singh vs. Walt Disney* June 2020). The appellate court reversed the trial court's assessment of property value based upon lack of evidence; however, it did uphold the lower court's decision to reject the valuation methodology, the Rushmore Approach, utilized by the Orange County Property Appraiser for its assessment of the Disney Yacht & Beach Club Resort.

In a 1978 monograph, Rushmore posited the first accepted methodology for separating income attributable to business (*intangible asset*) and income attributable to personal property from the entire income stream of a lodging facility (Rushmore 1978). The procedure, which to this day continues to reflect the thinking and actions of hotel-sector market participants, has been termed the Rushmore Approach.

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The appellate court ultimately agreed with Disney and the lower court by categorically rejecting the challenged assessment methodology utilized by the property appraiser in its valuation of hotel properties. Although the Rushmore Approach is used by other Florida county property appraisers, the appellate court opinion declared that, "the Rushmore Approach violates Florida law because it does not remove the nontaxable, intangible business value from an assessment."

Highlights of the appellate court's written decision are as follows:

Rushmore Includes Value of Intangible Business Assets. "We agree with the trial court that Appraiser, by using the Rushmore method, impermissibly included the value of Disney's intangible business assets in its assessment. The Rushmore method requires franchise and management fee expenses to be deducted from the total property income, which purportedly removes the business value from the assessment. However, it does not provide for adjustments to the gross business income for intangible business value prior to making those expense deductions. Jones testified that the deductions for franchise and management fee expenses removed all intangible business value, such as cash/working capital, favorable operating licenses, assembled workforce, brand, copyright, and goodwill. By taking a percentage out of a business's net income for management and franchise fee expenses, without first removing intangible business value from that gross income stream, the Rushmore method does not remove all business value from an assessment; to the contrary, we conclude that the Rushmore method ignores the fact that an intangible business value may be directly benefiting a business's income stream." (*Singh vs. Walt Disney* June 2020)

Rushmore Does Not Remove Nontaxable, Intangible Business Value. "Accordingly, we conclude that the Rushmore method violates Florida law because it does not remove the non-taxable, intangible business value from an assessment. Thus, the trial court did not err in rejecting Appraiser's ancillary income figure, derived using the Rushmore method." (*Singh vs. Walt Disney* June 2020)

Reassessment Would Include Nontaxable Assets. "On remand, Appraiser should not reassess the Property using the Rushmore method. As explained, Appraiser's assessment of ancillary income, conducted using the Rushmore method, failed to present competent evidence as to deductions for the intangible business value of Disney's operations on the Property. If Appraiser conducts a re-assessment using the Rushmore method, its assessment will yet again include these non-taxable assets." (S*ingh vs. Walt Disney* June 2020)

Opinion Reverses Trial Court. "While we would have preferred drafting an opinion that would resolve the parties' dispute, we find the record evidence is insufficient for us to do so. Accordingly, we reverse and remand to the trial court, with instructions that it remand to Appraiser for a reassessment of the Property consistent with this opinion." (*Singh vs. Walt Disney* June 2020)

Appellate Court Calls for Revised Assessment

The appellate court instructed that the assessment of the Disney Yacht & Beach Club Resort utilize an income approach analysis that considers hypothetical rental rates for ancillary revenue sources such as restaurants, bars, meeting/convention space, retail stores, parking facilities, and spas at the subject property. Essentially, the decision implied that including this type of ancillary income (aside from rooms) in the projection of total hotel revenue includes a business enterprise component that would overstate the value of the real property.

For example, the court is suggesting that in establishing the market value of a lodging facility, the actual revenue generated from hotel restaurant(s) and lounge(s) selling food and beverages is not the appropriate income to consider. Rather, the revenue should be speculatively established by what the real property (land, buildings, fixtures, and all other improvements to land) would supposedly rent for, established by competing restaurants, bars, meeting/convention space, retail stores, parking facilities, and spa properties in the market.

In addition to representing a fabricated rental analysis, which more than likely is not the highest and best use of the subject hotel property, under almost all circumstances, this rental rate consideration results in a manufactured relative lower revenue and net operating income, and a resultant market value conclusion that is artificially low.

Appraiser Requests Second Hearing

On July 2, 2020, the Property Appraiser formally requested another hearing with the Florida Fifth District Court of Appeal to reconsider the June 19 conclusion that "… the Rushmore method violates Florida law ... ." The appraiser indicated that if the appellate court would not reconsider, the matter would be brought to the Supreme Court of Florida.

On August 7, 2020, the Florida Fifth District Court of Appeal issued a revised opinion of the case, which concluded that similar to the trial court, the appraiser had incorrectly applied the Rushmore Approach (*Singh vs. Walt Disney* August 2020). However, the revised opinion did not declare that the Rushmore Approach itself violates Florida law.

The June 19, 2020, decision was perceived by many advocates as a tremendous victory for owners of Florida hotels and, ultimately, titleholders of lodging properties across America. Although this was not the first time that a court of law had ruled against the Rushmore Approach, in the context of real property tax, for decades the Rushmore Approach has been embraced by tribunals throughout the nation.

EHP Glendale, LLC, et al. vs. County of Los Angeles

In *EHP Glendale, LLC, et al. vs. County of Los Angele*, the county appealed a trial court summary judgment order finding that the valuation methodology used by the assessor, and accepted by the Los Angeles County Assessment Appeals Board, to value a hotel property was contrary to California law because it failed to exclude the hotel's intangible assets from the real property tax assessment. In February 2011, the Court of Appeal reversed on grounds that summary judgment was inappropriate when less than the entire Board administrative record was before the trial court when it entered its summary judgment order.

However, the Court of Appeal nevertheless addressed the issue upon which the trial court decision was based, that is, whether the Assessor's hotel income approach valuation was legally flawed. The Court of Appeal disagreed with the trial court's finding that the appropriateness of the assessor's value methodology presented an issue of law. According to the Court of Appeal, the assessor's income approach method was valid and presented only a question of fact as to its application. Invoking the presumption of correctness, the court found there was substantial evidence to support the Board's decision and remanded the case to the trial court for trial (*EHP Glendale vs. County of Los Angles* 2011).

Second Decision on *EHP vs. County of Los Angeles*

On September 18, 2013, the Second District Court of Appeal issued a second decision. On remand of the first EHP decision, a new trial court judge applied both the de novo and substantial evidence standards in ruling the assessor's and Board's income approach methodology was valid as a matter of law, and that substantial evidence supported the Board's decision. The hotel owner once again appealed, claiming that the value method applied was flawed as a matter of law. The second EHP court affirmed, finding its prior decision in EHP constituted the law of the case, which it was bound by law to follow unless the law was altered by an intervening decision by a higher tribunal *(EHP Glendale vs. City of Los Angeles* 2013). On December 18, 2013, the California Supreme Court denied the hotel owner's petition for review; however, in doing so it also ordered that the second EHP decision not be published or cited as authority.

SHC Half Moon Bay, LLC v. County of San Mateo

In May 2014, the Court of Appeal of the State of California, First Appellate District, Division Five ruled in *SHC Half Moon Bay, LLC v. County of San Mateo*, relative to SHC's (owner of The Ritz-Carlton, Half Moon Bay) claiming the assessment methodology was invalid because the assessment included nontaxable intangible assets. While the court ruled that the deduction of a management and franchise fee from the hotel's projected revenue stream was proper to remove the value of the hotel's intangible assets from the real property assessment, the ruling states that it did not entirely identify and exclude all intangible assets *(SHC Half Moon Bay vs. County of San Mateo* 2014).

Chesapeake Hotel LP vs. Saddle Brook Township

Chesapeake Hotel LP vs. Saddle Brook Township (Tax Court of New Jersey Docket No. 001690-99) was a seminal case decided on October 26, 2005. Highlights of Judge Peter D. Pizzuto's written decision include the following:

Court Accepts Rushmore Approach

"In Glenpointe (1989 New Jersey tax court case Glenpointe Associates vs. Township of Teaneck), the court accepted the conclusions of an expert appraisal witness, Stephen Rushmore, concerning the particular adjustments that are necessary to extract nonrealty income from total income so as to compute the income to be capitalized into real estate value." (*Chesapeake Hotel LP vs. Saddle Brook Township* 2005)

Income Attributable to FF&E Excluded from Realty Income

"Rushmore considered that all payments to the entity that manages and operates the hotel constitute business income generated by the exercise of management and entrepreneurship. Accordingly, he excluded these payments in the computation of realty income subject to capitalization. In addition, Rushmore considered that a portion of the overall income was realized by the employment of furniture, fixtures, and equipment (often referred to as FF&E). Since these items are (generally speaking) personal property rather than real estate, the income attributable to them, under Rushmore's method, is also excluded from realty income. Separate adjustments are made to provide for the periodic replacement of the personal property (the return of FF&E) and also for a yield on the investment in personal property (the return on FF&E). This method has been employed by experts in other hotel valuation cases and followed in reported decisions in New Jersey and other jurisdictions." (*Chesapeake Hotel LP vs. Saddle Brook Township* 2005)

Speculative Methodologies Abound

During the past 40 years, much has been written on what is commonly referred to today as "total assets of the business" or "business enterprise approach," and how the concept relates to lodging facilities. Unfortunately, most of what has been esoterically posited has been baseless and unsubstantiated by "the market" and has been put forth, for the most part, by generalist professionals who have no hotel educational background; little, if any, hands-on hotel operational experience; and little, if any, hotel investment expertise.

Essentially, these speculative methodologies are merely hypothetical academic constructs without any market foundation that have been developed for advocates for the sole purpose of reducing hotel property tax burdens. Analysis of the actions of hotel investors, however, indicates that the purchase of a hotel property reflects the acquisition of real and personal property only. Hotel investors account for income attributable to the business through the expense deduction of management and franchise fees. An investor purchasing a hotel "unencumbered" by a management agreement will not pay for a seller-assembled work force, business name, patents, copyrights, working capital and cash, operating procedures, and manuals. A passive investment in a first-class hotel "encumbered" by a long-term hotel management agreement is riskier, but no different than a passive investment in a class A office building occupied by a long-term creditworthy tenant. Either passive investment yields a risk adjusted return on property and not a business.

Conclusion

There can be only one market value, and the method employed to determine such must be the same under any circumstance, including property tax appeals. In other words, the assets and the rights being valued do not change simply because the valuation approach to an appraisal varies. Knowledgeable hotel investment market participants including buyers, sellers, lenders, and intermediaries do not acknowledge the existence of, nor ascribe a separate value to, intangible asset(s). Other than a deduction for management and franchise-related fees, this element is not reflected in their underwriting and investment decisions.

It is only property tax advocates that utilize "total assets of the business" or "business enterprise approach" and for the sole purpose of attempting to achieve reduced hotel property assessment valuations. If "total assets of the business" or "business enterprise approach" were utilized for mortgage debt purposes, hotel real property market values could be highly deflated because an inordinate amount would be allocated to unfinanceable intangible assets, and result in greatly diminished debt proceeds and existing loans deemed underwater.

The issue of the appropriate treatment of hotel intangibles has engendered much confusion and discussion among appraisers, assessors, attorneys, judges, lenders, and regulators. The matter has been over-complicated for nothing, and way too much time has been spent debating this issue. The fact is that the Rushmore Approach has endured because it reflects the thinking and actions of hotel investment market participants. Until the market alters its underwriting/pricing of hotel assets, there is no justification for accepting unfounded "violation" and/or "illegal" judicial property tax valuation rulings promulgated by courts of law that know little, if anything, about real estate and/or hotels.

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Blog

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